Department of Mathematics and Statistics
University of Waikato

A.G. Schaake
J.C. Turner
D.A. Sedgwick

BRAIDING

– REGULAR KNOTS

A Series of Books on Braiding

Book 1/1 * * * * * 1988

New and Automatic Construction Methods

A Series of Books on Braiding

Book 1/1

Published by the Department of Mathematics and Statistics,
University of Waikato, Hamilton, New Zealand.

BRAIDING

NEW AND AUTOMATIC METHODS FOR CONSTRUCTING KNOTS AND BRAIDS

A. G. SCHAAKE, *Waikato Polytechnic*
J. C. TURNER, *University of Waikato*
D. A. SEDGWICK, *Waikato Polytechnic*

BOOK 1/1 — REGULAR KNOTS

A SERIES OF BOOKS ON BRAIDING

1988

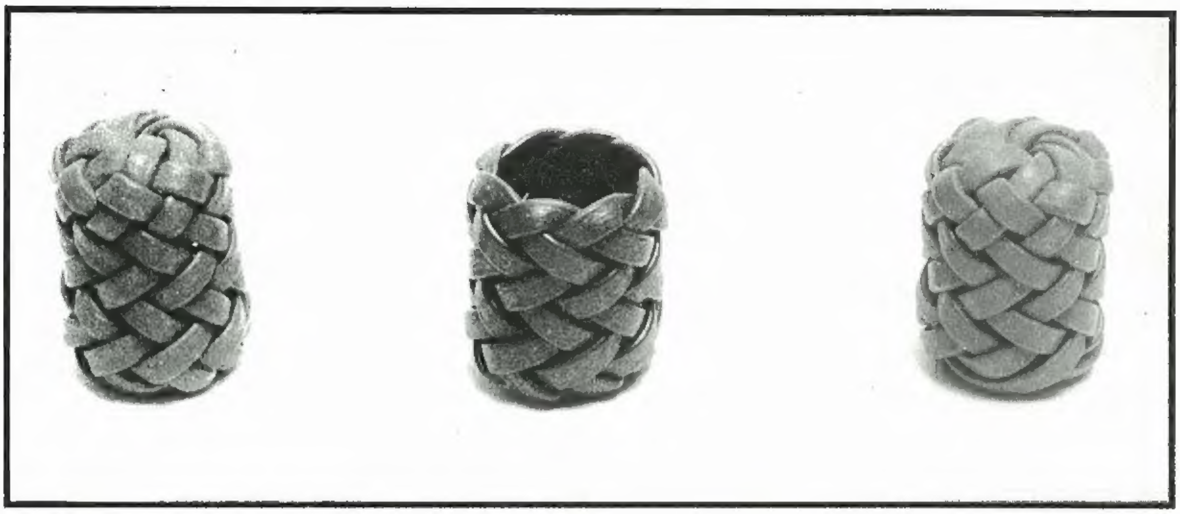
First Edition 1988

ISBN 0-908830-00-9 (set)
ISBN 0-908830-01-7 (v.1)

CONTENTS

Regular knots on facing page

(i)	*Aztec-Fan Knot (15/8)*	*Ex.29*
(ii)	*Fan Knot (11/8)*	—
(iii)	*Aztec-Scallop Knot (15/8)*	—
(iv)	*O3 Knot (16/9)*	*Ex.41*
(v)	*Basket Weave Knot (17/15)*	*Ex.36*
(vi)	*Fast Helix Knot (17/10)*	*Ex.35*
(vii)	*Slow Helix Knot (17/10)*	*Ex.34*
(viii)	*Slow Helix Knot (22/15)*	*Ex.33*
(ix)	*O1 Knot (17/10)*	*Ex.38*



(I) **(II)** **(III)**

(IV) **(V)** **(VI)**

(VII) **(VIII)** **(IX)**

Knots are more numerous than the stars;
and equally mysterious and beautiful ...

FOREWORD

This book is the first of a series which will present new theories and methods of braiding. Each book will treat one or more types of braid, and will give methods and formulae which will enable artisans to produce similar braids of their own design. The methods are based upon mathematical principles discovered by Georg Schaake in recent years.

Many examples and exercises will be given, to give readers chance to practise using the methods and become familiar with computing the weaving algorithms. The methods are generally very easy to learn and apply, once the underlying principles of the arithmetic calculations are grasped.

We shall not give practical details on braid production, such as what kinds of tools and thongs to use. Readers requiring this kind of information are referred to the excellent encyclopaedia of rawhide and leather braiding written by Bruce Grant (see the bibliography for details and other references).

For desired braids of small numbers of parts and bights, the step-by-step weaving formulae can easily be computed 'by hand'. For larger braids with complex designs, it may be preferred to use a computer. Computer software and diskettes for personal computers will be made available. The software for this book will provide the braider with the fully tabulated braiding algorithms for all desired Turk's Head Knots, Ring Knots, Gaucho Knots, Head Hunter's Knots, Perfect Herringbone Knots, Slow Helix Knots Type II and members of families of Fan Knots; all these up to any required bight and part numbers. All the braider will have to do to obtain a desired tabulated braiding algorithm is to enter values for two or three knot parameters. The diskette will also contain the necessary software for obtaining the tabulated braiding algorithm of any regular knot of the braider's own design. These diskettes will be obtainable from the publisher.

We wish to record our gratitude to Janet Smith and Erica Harris, who have given so much help in the typing of the many drafts of this document and in the preparation of the final one in computer type-setting form using TEX.

A.G.S.
J.C.T.
D.A.S.

June, 1988

1

INTRODUCTION

Knotting and braiding are two skills that men and women have practised and developed since time immemorial. Both have been, and are still, of immense practical value to mankind; and both can be applied to produce articles of great artistic beauty.

Drawings found in caves inhabited by prehistoric man, dating back 15,000 years and more, tell us that even then our ancestors knew how to use the skins of the animals they had killed. It seems clear that they knew how to prepare the hides in order to make clothes and blankets to keep themselves dry and warm. They would know too how to make strings and twines, perhaps ropes, using naturally growing fibres, and how to use these for lashing objects together. Leather thongs would also be used for knotting and for sewing skins together and producing garments. Later, as time passed through long centuries, they discovered how to preserve animal hides by chemical processes, finally evolving the tanning methods which are used today.

Many and varied are the practical uses to which strings, ropes and leather skins and thongs have been put. Indeed, the livelihood, safety, comfort and well-being of most (perhaps all) mankind has depended to a very great extent on abilities to produce useful artifacts with the aid of knotting and braiding materials and skills.

Today we are hardly less dependent on these artifacts and skills than were our earlier, more primitive brothers and sisters, even though knots and braids are less clearly in evidence in our sophisticated modern surroundings.

Quite apart from their practical uses, knots and braids have many artistic possibilities. *Fancy knotting* and *decorative braidwork* have long been recognised as two of man's folk arts. The sailor on his full-rigged ship, with the miles of rigging and the countless, varied knots that had to be tied to form and maintain them, developed knotting into an art that was peculiarly his own. Likewise, the American *cowboy*, the Mexican *vaquero* and the Argentinian *gaucho* all used leather extensively in their work. They were expert *trenzadores*, or braiders; and many brought the art of leather decoration on their own equipment and that of their horses to high peaks of attainment.

Excellent accounts of the history, uses and construction of knots and braids may be found in the following two encyclopaedias:

Encyclopedia of Rawhide and Leather Braiding, B. Grant (1972);
The Ashley Book of Knots, by C.W. Ashley (1944);

In this series of books the symbol [n] will be used to refer to a book or article which is described in the bibliography at the end of a book and labelled there by the integer n. For example, in this book [2] will refer to The Ashley Book of Knots.

Between them the encyclopaedias [1] and [2] contain perhaps ten thousand drawings and working diagrams, representing some four to five thousand knots and braids. By following the diagrams carefully, with string or leather thong in hand, and reading the instructions given alongside, a reader may successfully learn to produce the various knots. Moreover, the reader will find that the thousands of knots depicted have been grouped into chapters, or sections, which goes some way towards providing a rough classification of the *universe* of all knots into *types* or *subclasses* of knots. For example, in [1] there are sections headed Twist Braids, Slit Braids, Flat Braids, Round Braids, etc., etc., which do organize braids in a valuable way. However, this is not a scientific classification. As a division into types it has many failings, inconsistencies, duplications, even errors. The scientific or mathematically-minded reader of these encyclopaedias might wonder why the whole treatment of the formation of knots has not been placed (and long since) upon a surer footing than the graphical, wholly empirical, one that is followed by Grant and Ashley. Their works are monumental, of great beauty, worth, and scholarship; but they espouse no systematic method for producing knots and braids, no theoretical treatment. Among the very few references to mathematics in these works, the following two are revealing: [1] (p.440) and [2] (p.233). Below we reproduce the [1] reference in full. It occurs in a two-page section headed *The Braiding Detective*; it gives the only clues to the reader that in the wonderful and infinitely extensive world of braiding there lies the possibility of a theory of braiding, and of methods whereby the artisan can develop and analyze his own creations of decorative braids.

> "**Duplicating Strange Braids and Creating New Ones.** The braided knot, or even the Turk's-head, is a mathematical marvel. It goes round and round and comes out perfect — the working end of a braided knot returns to its point of origin.
>
> It is interesting to analyze, more absorbing to create, and definitely a challenge. Distinct variations are limitless and all of them are adjusted to a mathematical scale of easy deduction.
>
> Dr. Almanzor Marrero y Galíndez, in the prologue of his book, *Cromohipologiá*, said this of his Argentine father: 'During his last years almost each day he made an original *criollo* button, which he first posed as a problem, resolving it mathematically, and later executing the formulae he had worked with success. Some of these buttons were of such complexity that he had to use several strings in the same knot, whose total length reached several meters.'

> Alas, his mathematical techniques in planning his buttons have been lost to posterity. Many serious scientists and mathematicians have given profound thought to the Turk's-head and as an independent result of the studies of Clifford W. Ashley of New England, and George H. Taber of Pittsburgh, the 'Law of the Common Divisor' was discovered."
>
> — [1], p.440 (B.Grant)

The *law of the common divisor*, as enunciated by Ashley in [2], p.233, is as follows. It refers to single-string turk's head knots:

> *A knot of one string is impossible in which the number of parts and the number of bights have a common divisor.*

This would seem to be the extent of any known mathematical theory of the *process* of braiding. Moreover, that law was arrived at empirically. No proof was given for its general truth. There appears to be no extant mathematical theories (certainly Grant and Ashley did not use any) which enable *algorithms* (i.e. step-by-step methods) or *recipes* to be calculated and written down so that artisans can produce braids of given patterns. Without such a theory, of course, fundamental questions about the universe of braids cannot even begin to be answered. For example, we might ask: "Suppose we require to produce a braid with a certain herringbone pattern of under and over string-crossings, using only one string and having 95 parts and 36 bights. Can it be done ? If so, *how* may it be done ?" The encyclopaedias of Grant and Ashley give no clues for answering such questions.

There *is* a highly developed mathematical theory for studying knots and braids, which began in the mid-to-late nineteenth century with the work of Gauss, Listing, Kirkman and Tait. all great mathematicians (*very* great in the case of Gauss). In the early twentieth century the study of knots became essentially combinatoric and topological. Work of men such as Dehn, Alexander, Artin and Fox led to extensive theories of knot invariants, which were based largely on group theory applied to groups of loops in the spaces that surround knots (see [7], for example).

None of these theories, however, has anything to say about the actual tying of knots and braids. Indeed, a topological knot theorist might not be interested at all in the kinds of problems that would intrigue a braider (e.g. the 95 part-36 bight question asked above). To emphasise this point, using a different type of example, let us consider a reef knot and a lark's head knot. In the knotter's world these are different knots, with different ways of tying them. But because one can be transformed into the other by a few simple moves of the string, a topological knot theorist says they are equivalent knots, and has no further interest in examining their differences.

The first named author of this book, Georg Schaake, is both an engineer and a person deeply interested in the crafts of leatherwork and knotting. Over a period of eight and more years he has been seeking to fill the gap which has been exposed above, namely the total lack of any general mathematical theories which model the processes involved in knotting and braiding. In this period he has developed many relationships between numbers of parts (p) and numbers of bights (b) in a braid, and these relationships form the basis of a beautiful theory of braiding. It turns out that this theory is a branch of

number theory; the models of braids that he has discovered relate directly to solutions in integers of the linear equation $ax + by = c$, and through this to well-known theories of continued fractions. We may properly attach the adjective 'diophantine' to much of his modelling, and thus distinguish this new kind of knot theory from the classical topological kind.

By means of his findings, Mr. Schaake has been able to devise many simple algorithms (recipes) for the production of braids of given types and decorative pattern. Further, the classification of braids can now be attempted with a sound rational basis to underpin it.

In 1987 Mr. Schaake joined with Dr. John Turner, who is a topological knot theorist, to study relationships between the two kinds of knot theory and to determine how Mr. Schaake's ideas may best be disseminated both to the world of the knot theorist and to the world of the craftsman and the artisan — the producers of decorative knot and leather work. They decided that it was necessary to produce a series of books, each dealing with a particular type or class of braids. This book, the first of the series, treats the class of regular knots.

The third author, David Sedgwick, is helping to develop the braiding algorithms by carrying out computer-aided studies; he is also writing software which will enable artisans to obtain for themselves algorithms for the artistic braids which they want to produce.

In this book the reader is introduced to the new theories and methods in very gentle fashion, beginning with a section entitled Basic Ideas. This presents a method and diagrams for producing a simple 2-part braid by making a series of passes of a string around a cylinder. Many readers will be familiar with this procedure, but they are advised to follow the method carefully and read all the instructions, for in doing so they will learn not only the notation and terminology but also important differences in emphasis or action-sequences from procedures such as are given in [1] and [2]. They will also become acquainted with a type of grid-diagram which constitutes a complete model of the braid being tied, and which is used consistently to describe all other braids and knots in the series of books.

The level of mathematics used in the book has been kept as low as possible. Some algebraic notations have had to be introduced, in order to describe the algorithms being presented. These have been defined at the points of first use, and examples given to make their meanings clear. Those readers with mathematical training who wish to examine proofs of the algorithms and theoretical results on braiding, may consult the research reports which are listed under RR1/1 and RR1/2 in the Bibliography. Copies of these may be had by writing to Dr. J.C. Turner, Department of Mathematics and Statistics, University of Waikato, Hamilton, New Zealand.

A collection of Exercises, to help readers come to a good understanding of the notations and algorithmic methods, is included at the end of the book, and full answers are supplied.

Books which are in preparation to follow this one are:

1/2	Semi-regular knots.
2/1	Fiador knots.
3/1	Herringbone knots.
4/1	Standard pineapple knots.
4/2	Perfect pineapple knots.
5/1	Grant knots.
6/1	Starting and finishing methods in braiding.
7/1	Integrated knots.
8/1	Inter-braiding techniques.
9/1	Transition braids, I.
9/2	Transition braids, II.
9/3	Transition braids, III.
10/1	Long knots.

The authors would be pleased to receive comments from readers on the new methods given in the books. Suggestions for improving the manner of their presentation would also be welcomed.

2

BASIC IDEAS

In this section we describe many of the notations and processes that we shall need later when explaining our general methods for obtaining braiding algorithms. Our aim is to show how a braid can be constructed by passing a string from left-to-right, right-to-left, and so on, round and around a cylinder, interweaving the string with itself by making a succession of under- and over-crossings as the passes are made.

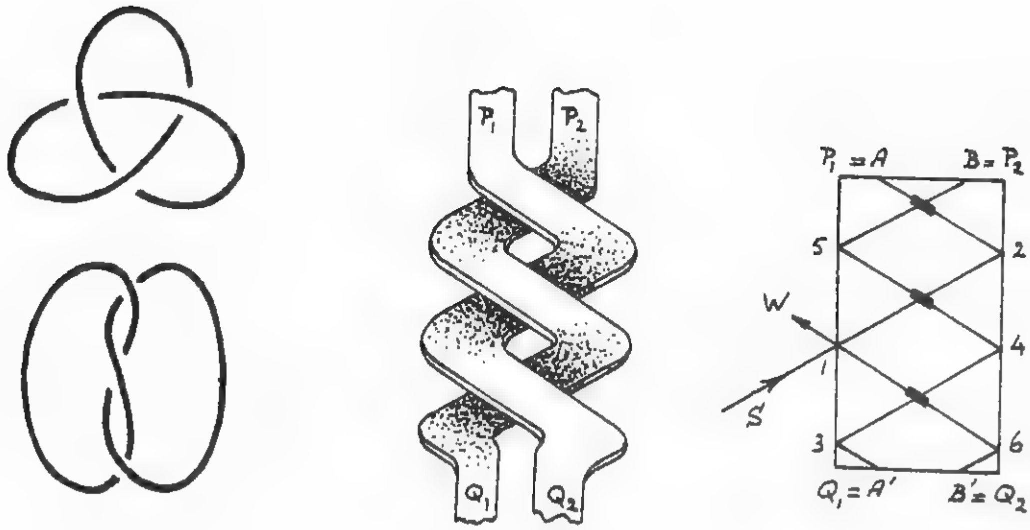
The essential concepts to be learned are the process itself, how a certain grid-diagram represents the braid (and the process), and how an algorithm (that is, a step-by-step method) can be obtained which gives a list of instructions for carrying out the interweaving as the string passes round and around the cylinder.

Terms that will be defined include *parts, bights, string run, coding, grid diagram,* and *algorithm table.*

Our explanations and definitions are given with reference to a very simple and familiar 3-crossing knot, namely the *trefoil braid.*

The knot construction process

Let us begin by showing diagrams of a 2 part—3 bight turk's head knot (in the classical theory called the trefoil knot), a 2 string—3 bight under/over braid (trefoil braid) and a 2 part—3 bight turk's head knot pictured after being "tied" around a cylinder in the manner to be described.



trefoil knot.

(a)

2 string—3 bight u/o braid.

(b)

regular knot 2 part—3 bight turk's head knot.

(c)

FIGURE 1. *Diagrams of a three-crossing knot and braid.*

The diagrams (a) and (b) are well-known representations of the right-handed "trefoil knot" and braid respectively. Note that (a) is formed from a single closed string; whereas (b) is formed from two interwoven strings, one running from point P_1 to Q_2, and the other from P_2 to Q_1.

Next note that if a circular cylinder be placed *behind* the braid (b) with its straight-line generators parallel to P_1P_2 and Q_1Q_2, then the braid can be wrapped backwards and around the cylinder until P_1 meets Q_1 and P_2 meets Q_2.

Let us assume this done, and that we have joined the ends P_1 and Q_1, and also P_2 and Q_2. We observe that we now have the one-string 2 part—3 bight knot (trefoil knot) tied around the cylinder. If we slip it off the cylinder, moving carefully in the direction $P_1 \to P_2$, we can then arrange it on a flat plane in the manner of diagram (a).

Now examining diagram (c) we see that it provides a clear picture of the 2 part—3 bight turk's head knot (trefoil knot) tied around a cylinder. The thick, short line-segments indicate where one string has crossed over another. In fact the diagram tells us much more: it provides full information as to how the *process* of tying the knot on the cylinder must be carried out if a 2 part—3 bight turk's head knot (trefoil knot) is to be achieved. To be specific, *diagram (c) is a complete visual algorithm for constructing the right-handed trefoil knot.* We shall call it the *grid diagram* for the knot. Let us analyze this algorithm by identifying six steps that have to be taken to complete the knot. The reader would best understand the algorithm if he or she were to take a wooden cylinder some two inches in diameter, a length of string, and two elastic bands to keep the string bights in place, and actually carry out the steps as described. Diagrams to illustrate the steps are given later, on pages 14 and 15.

Algorithm

Step 1: L → R upwards; run 1 to 2; no crossings.
Step 2: R → L (right to left); run 2 to 3; no crossings.
Step 3: L → R (left to right); run 3 to 4; no crossings.
Step 4: R → L; run 4 to 5; cross *over* 1–2.
Step 5: L → R; run 5 to 6; cross *under* 2–3.
Step 6: R → L; run 6 to 1; cross *over* 3–4.

If finally we join the working part (W) to the standing part (S) at 1, then we shall have completed the 2 part—3 bight turk's head knot (trefoil knot). (N.B. to get it off the cylinder it will be necessary to cut and remove the elastic bands; but that is incidental and irrelevant to the algorithm. In principle the elastic bands can be dispensed with.)

Observations

(i) In diagram (c) we indicated where the changes in direction of the string occur by placing numbers 1, 2, ..., 6 upon them. This numbering is unnecessary. However, it is pertinent to observe that all odd-numbered changes occur on the left-hand elastic band, and all even-numbered ones on the right-hand band. A knot cannot be completed unless (1) there is an even total number of changes of direction and (2) the working part returns to the standing part (where it may be joined).

Suppose a knot has $2n$ changes of direction in all. If we were to cut the string at all the n even-numbered change-points, the knot would split into n equal pieces like

We call such pieces *bights*. Thus a 2 part—3 bight turk's head knot (trefoil) has 3 bights.

(ii) At each step the string *runs*, either from R → L or from L → R. In every such run, called a *half-cycle*, in the final form of the trefoil knot one crossing point is encountered which separates the half-cycle into 2 *parts*.

We say that the trefoil knot is a 2 part—3 bight knot (or a 2/3 knot).

(iii) The thick-line segments which are marked on the grid diagram (c) are to be interpreted as follows:

string run (left to right) goes *under* at the crossing.

string run (right to left) goes *over* at the crossing.

In later examples of grid diagrams we shall meet the other possibility for crossing marks, viz.:

string run (left to right) goes *over* at the crossing.

string run (right to left) goes *under* at the crossing.

(iv) Had we left out of diagram (c) the thick-line segments, we would have still been able to run the string around the cylinder, forming 3 complete bights in a well-defined way; but we would not have known whether to form under- or over-crossings, when we met existing strings with the moving or working part. Nevertheless, without any interweaving, we would have split each half-cycle into 2 parts, and therefore have obtained a 2 part—3 bight knot, which we call *the 2/3 string run.*

It is therefore important to note that in order to completely specify a knot "tied" on a cylinder one can proceed thus:

(1) specify the *string run*;

then (2) specify the under-over pattern; this pattern we call the *coding* for the knot.

Thus one diagram, as in (c) but without the marks on it, will specify a string run which forms the basis of many different knots according to which coding is selected to apply to it.

(v) The labels A, A', B, B' on diagram (c) indicate how the half-cycles 2–3 (i.e. 2–A–A'–3) and 5–6 (i.e. 5–B–B'–6) are completed as the string passes around the cylinder. Note that all L→R half-cycles are parallel and so are all R→L ones, in the completed knot.

Tabled Algorithm

The algorithm for producing the 2 part—3 bight turk's head knot can be arranged simply in tabular form as follows. We use the letters **u** and **o** to mean *under* and *over* respectively.

2/3 turk's head algorithm	
half-cycle	crossings
1. L → R	none.
2. R → L	none.
3. L → R	none.
4. R → L	o.
5. L → R	u.
6. R → L	o.

Diagrams showing how each step of the algorithm is carried out

To complete this introductory section, and we hope finally to make everything completely clear, we give six diagrams in Fig. 2 on pages 14 and 15, one for each of the steps listed in the algorithm table. If the reader will construct a trefoil knot, passing a string round a cylinder and checking out both the diagrams and the tabulated steps at the same times, the whole process and its tabular representation will become crystal clear. Once the trefoil knot is completed, the reader should check that it has indeed got 2 parts and 3 bights, hence its alternative name, the 2/3 turk's head knot.

Now that the basic terms and the braiding process have been explained, we are in a position to define a general class of braids and show how methods for tying them can be obtained. The remaining sections of this book treat the class of *regular knots*, which includes the turk's head knot and many other well-known types of cylindrical braid.

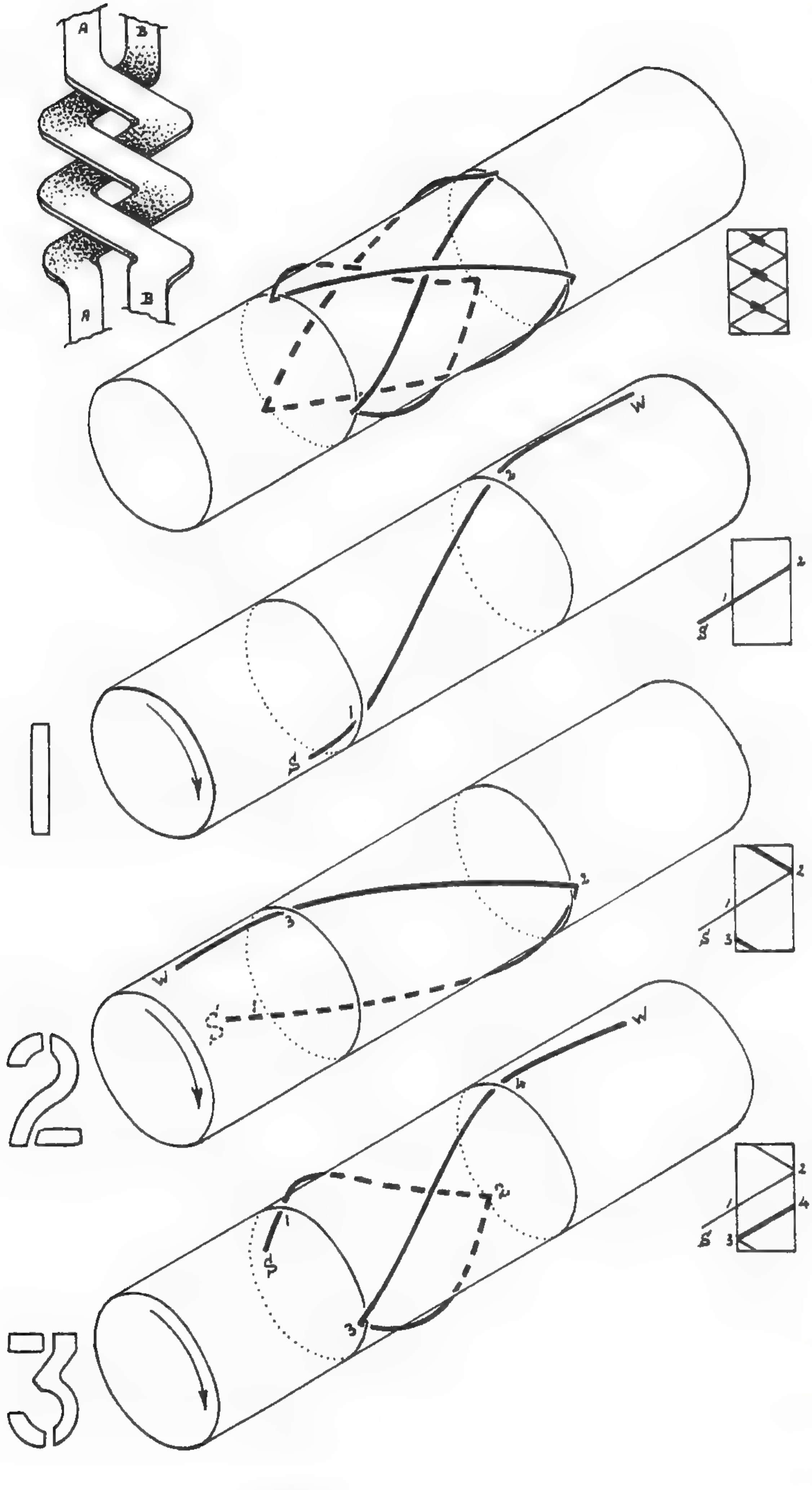


FIG. 2

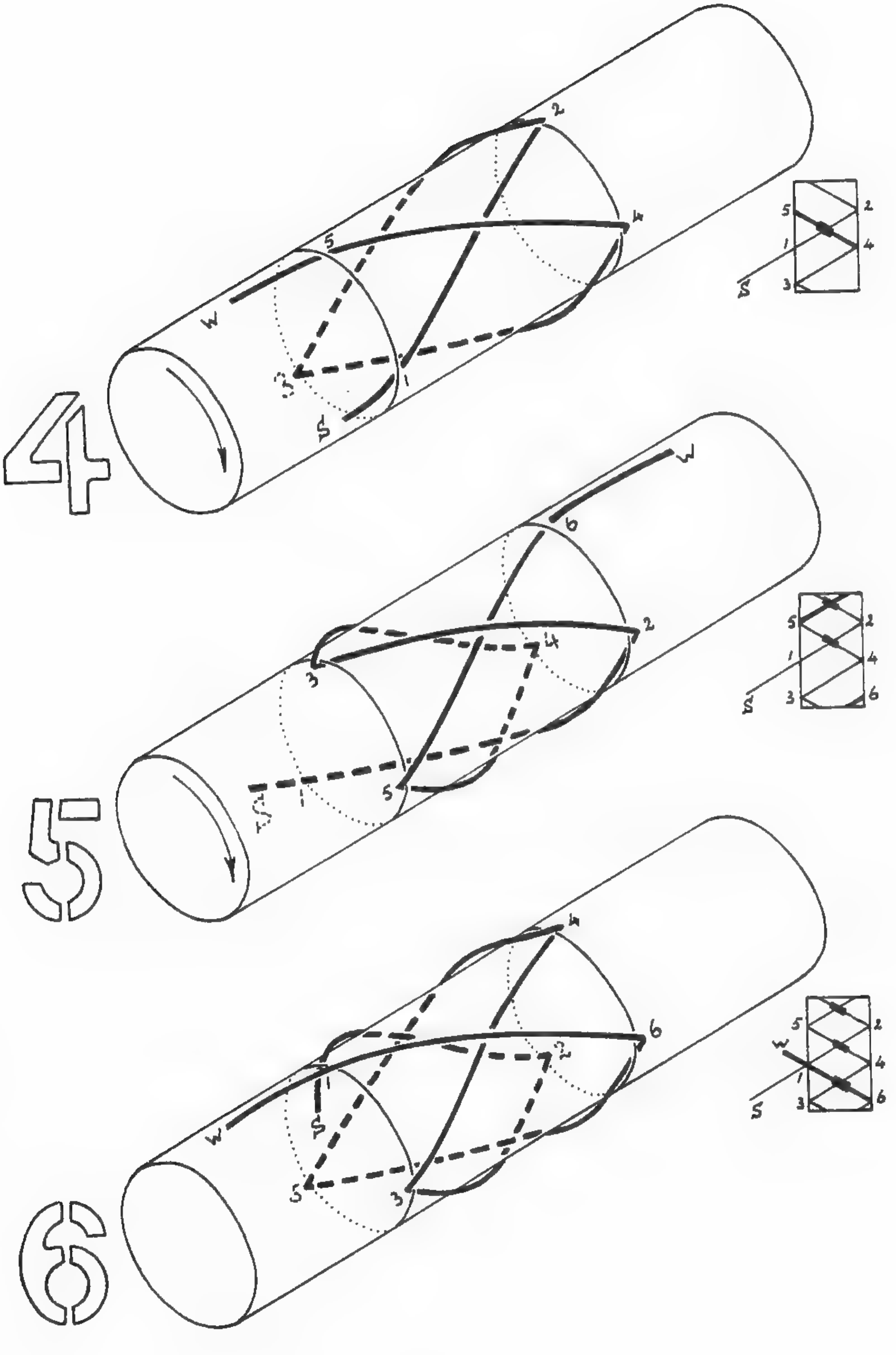


FIG. 2 CONT.

3

THE REGULAR KNOT

Each of the Figures 3–1, 3–2 and 3–3 on the next page represents a *regular* flat braid of seven strings and five bights. A *regular* flat braid is a flat braid having all its left hand bights on a single straight line and at the same time all its right hand bights on another parallel single straight line. It consists of two sets of string runs, with all the members of a set being parallel and running from one bight boundary to the other.

Imagine that behind any of the braids is placed a horizontal cylinder and that we wrap the braid backwards around the cylinder, joining the top number 1 to the bottom number 1, the top 2 to the bottom 2, the top 3 to the bottom 3, . . . , and finally the top 7 to the bottom 7. The result is a regular cylindrical braid knot.

If we now follow any chosen string round and around the cylinder we shall find that after seven complete revolutions we return to our starting point. Thus the regular cylindrical knot can be made from one single string. Such *single* string knots, which can be thought of as being endless regular flat braids, are called *regular knots*.

The *string run* (see p.12) for each of the figures is the same; the patterns of crossings however, are different. We can regard a crossings pattern (i.e. the *coding run*) as being superimposed upon the string run.

Our definition of a class of regular knots has led us to use a narrower definition of turk's head knots than is normally used by braiders. We define a turk's head as follows:

> A *turk's head knot* is a regular knot having a super-imposed coding which alternates overs and unders throughout.

With this terminology the regular knots commonly described as *basket weaves*, are turk's heads. We also use the term basket weave, but for other kinds of regular knot; examples of these are given in the exercises (see Ex.36 and Ex.37) at the end of the book.

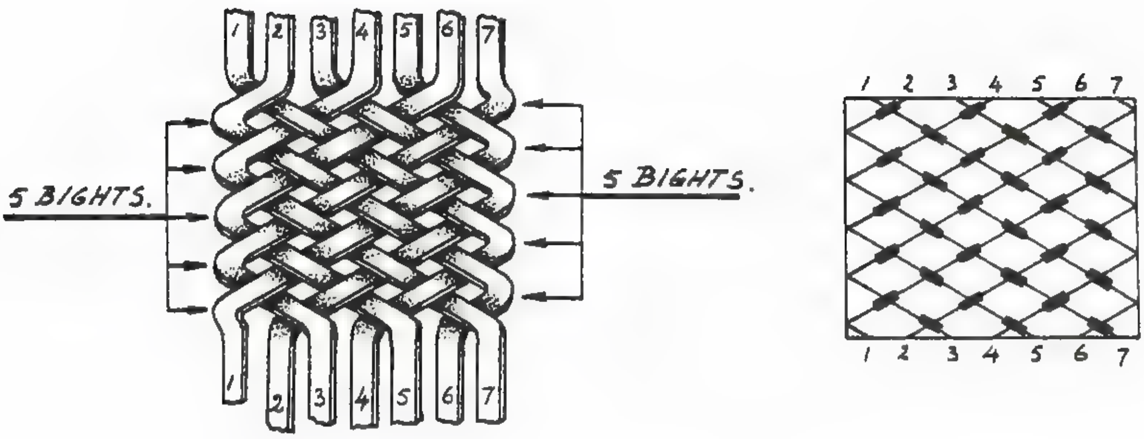


FIG. 3-1. TURK'S HEAD CODING.

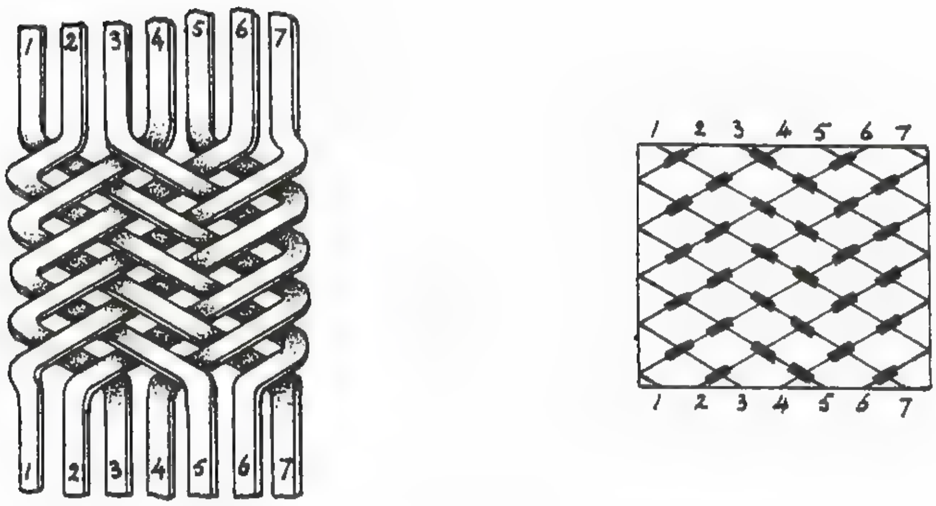


FIG. 3-2. TWO PASS HEADHUNTER'S CODING.

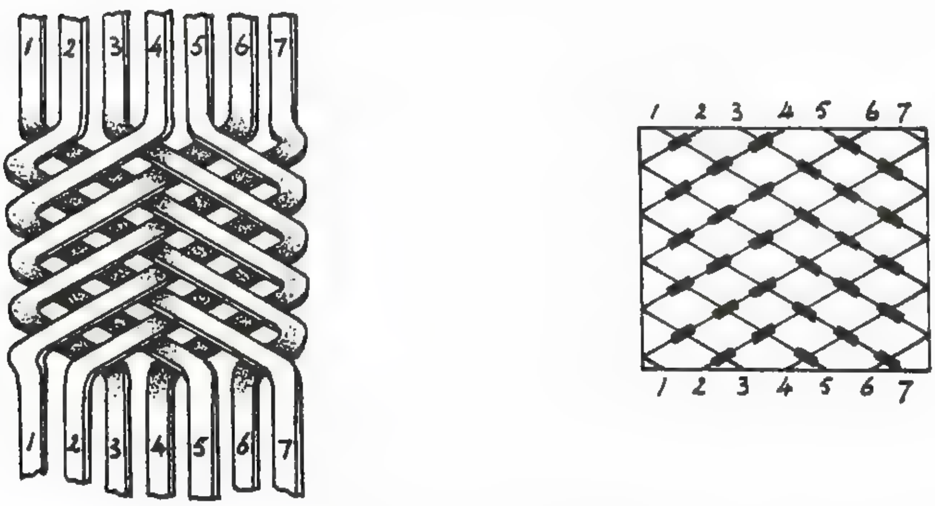


FIG. 3-3. THREE PASS RING KNOT CODING.

4

THE GRAPHICAL REPRESENTATION OF REGULAR KNOTS

Rules for the construction of any desired regular knot can be given in simple terms referring to the grid diagram which in effect represents the flat braid from which the cylindrical knot can be formed. We gave an example of a grid diagram for the trefoil knot as Figure 1(c) on p.10. Further examples of these grid diagrams appear in Figures 3–1, 3–2, 3–3 on page 17.

We shall explain the graphical representation of regular knots with reference to the 7 *part—5 bight turk's head* example (shown in Figure 3–1). The 7 part—5 bight (indicated as 7/5 or as p/b = 7/5) turk's head is a regular *cylindrical* knot. As has been said before all regular knots are *single string regular cylindrical* knots, but not all such knots are turk's heads. This can be seen from the Figures 3–1, 3–2 and 3–3, only the first of which is a turk's head.

If a single-string cylindrical knot is cut along a cylinder-generating line which lies between two *adjacent rows* of string intersections, and the cut cylindrical knot is then unrolled into a flat plane in *front* of the cylinder, a simple projection onto the plane provides the grid diagram. Over-crossings, where one string passes over another, can be indicated by short, thickened lines.

A convenient grid onto which to draw the representation of a regular knot is the *isometric grid* (Gormack graph paper : Christchurch N.Z. No. 0810 Isometric Grid).

Recall from the Basic Ideas section that every knot can be defined by specifying both its string run and its coding run. In Figures 4–1 to 4–11, page 21, all the details of the string run and coding run for the 7/5 turk's head are illustrated. Explanations and notes on these now follow. In the next Section, referring to Figures 5–1 to 5–10, a full description of the step-by-step construction of this same knot is given.

The developed string run of the cut cylindrical knot can be drawn on the grid as in Fig. 4–1 *or* as in Fig. 4–2. The top horizontal line is the same as the bottom horizontal line and is the line along which the cylindrical knot was cut. S is the standing part of the string and W is the working part. The 7/5 turk's head has 5 bights along the left boundary and 5 bights along the right boundary as indicated in Fig. 4-3; and it has 7 parts, as indicated in Fig. 4–4 or Fig. 4–5.

A string run from the left boundary to the right boundary, or from the right boundary to the left boundary, is called a *half-cycle*, and we see from Fig. 4–6 that the number of string intersections along a half-cycle is equal to one less than the number of parts, that is, $p-1$. All string intersections occur in vertical columns, numbered from left to right in Fig. 4–7 and numbered from right to left in Fig. 4–8. If desired, vertical lines can be drawn on grids to highlight the columns of intersections. We see from Figs. 4–7 and 4–8 (and clearly from 4–10 and 4–11) that the number of columns is equal to the number of parts. Starting at S the first half-cycle runs from 1 to 2, the second half-cycle runs from 2 to 3, the third half-cycle runs from 3 to 4, and so on, as indicated in Fig. 4-9. Thus the right hand bights have even numbers and the left hand bights have odd numbers; furthermore the total number of half-cycles is equal to twice the number ($b = 5$) of bights. When describing the knot, we quote $b = 5$ bights, although in fact the knot has 10 bights in all, 5 on each side.

The coding is superimposed upon the string run as in Fig. 4–10 for a turk's head coding.

The labelled grid diagram of Fig. 4–11 gives us a complete graphical representation of a 7/5 turk's head.

All regular knots are single string cylindrical knots having all the left-hand bights on the extreme left column and all the right-hand bights on the extreme right column and consisting of two sets of string runs, with all the members of a set being parallel and running from one bight boundary to the other.

The class of regular knots can be divided into three main subclasses:

(A) those which have a constant coding per column (*column-coded regular knots*);
(B) those which have a constant coding per row (*row-coded regular knots*);
(C) those not belonging to either (A) or (B).

By 'constant coding' we mean that all the intersections on a given column (or a given row) have the same crossing mark.

Note that the turk's head belongs to both subclasses (A) and (B). Thus the subclasses (A) and (B) overlap.

In Research Report 1/1 [RR 1/1] it is shown that for a *regular* knot the *number of parts* and the *number of bights* cannot have a common factor other than 1.

There is a class of knots which have all the properties of regular knots *except that* their numbers of parts and bights have common factors other than 1; as a consequence they are multi-string knots. These knots are called *semi-regular knots*; they are treated in Book 1/2.

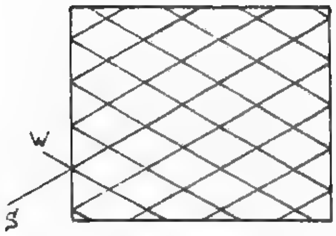


FIG. 4-1.

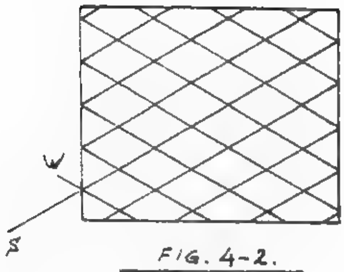


FIG. 4-2.

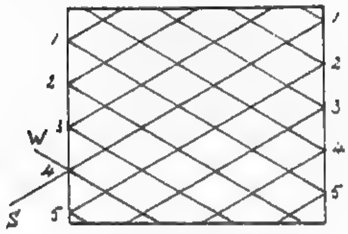


FIG. 4-3.

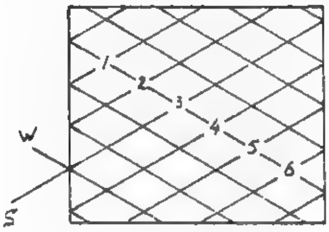


FIG. 4-6.

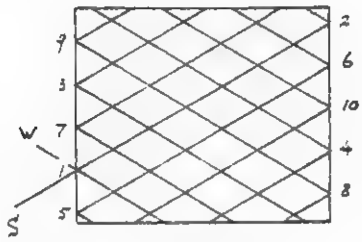


FIG. 4-9.

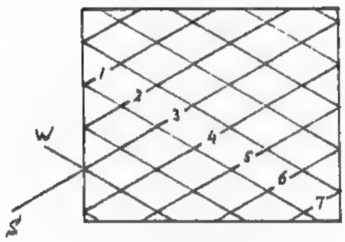


FIG. 4-4.

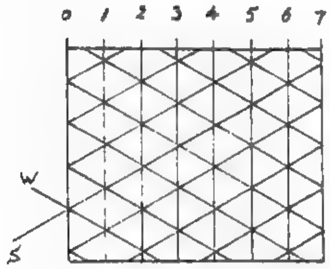


FIG. 4-7.

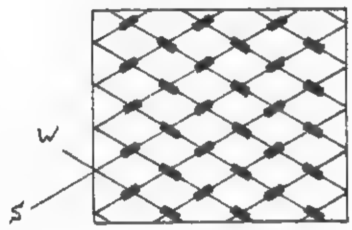


FIG. 4-10.

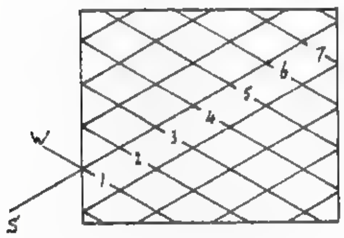


FIG. 4-5.

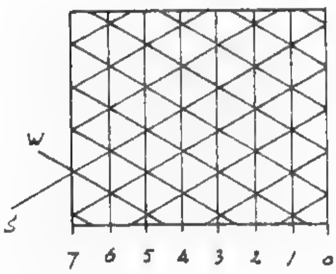


FIG. 4-8.

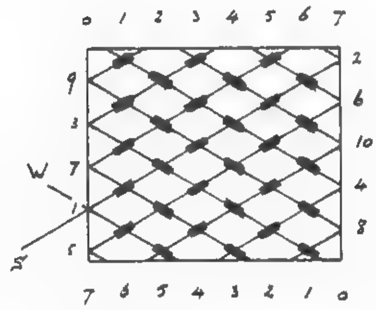


FIG. 4-11.

x OVER y.

y UNDER x.

x UNDER y.

y OVER x.

5

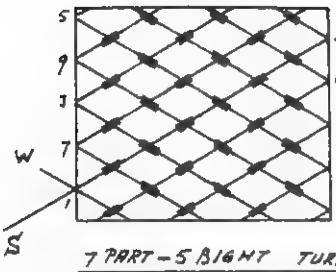
THE USE OF THE REGULAR KNOT DIAGRAMS IN THE BRAIDING OF THE REGULAR KNOT

In this Section we give practical details for braiding regular knots, and use as first example the 7/5 turk's head whose grid diagram was described in pages 19,20. We follow this with two other examples, giving diagrams of the steps for braiding both the 11/4 and the 4/11 turk's head knots.

Figure 5 represents the 7 part/5 bight turk's head in graphical form. Figs. 5–1 to 5–10 (next page) give a complete, step-by-step description of how the braid is tied by passing the string from left to right, then right to left, and so on around a cylinder. Details for constructing the braid are as follows.

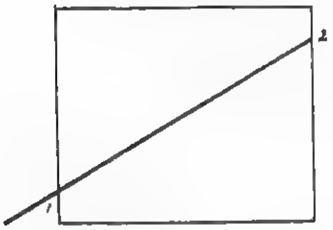
Take a piece of broom handle of about 200mm long (20cm; 8″); hold it horizontally in front of you and then put two elastic bands around it; one on the left and one on the right about 50mm (5cm; 2″) apart. Take a length of string, about 1 metre long, and slide one end under the left elastic band letting it project to the left for about 50mm; this is the *standing end S*. Looking from above rotate the stick for about three quarters of a turn towards you and feed the other end (this is the working end W) of the string under the right elastic band. The result is presented in graphical form in Fig. 5–1. Now turn the working end to the left over the right elastic band and rotate the stick again for about three quarters of a turn towards you; then lead the working end over the previously formed half-cycle (Fig. 5–1) and feed it under the left elastic band. The result is graphically illustrated in Fig. 5–2 (where the dotted line is the first half-cycle laid down and the solid line the just-completed last half-cycle.) Now turn the working

FIG. 5



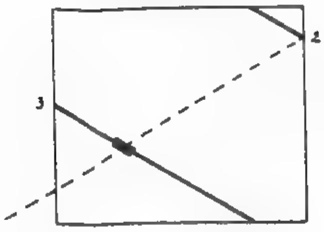
7 PART – 5 BIGHT TURK'S HEAD

FIG. 5-1.



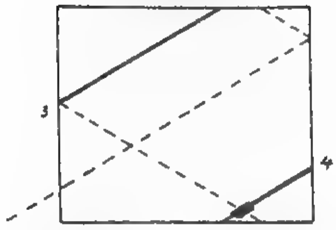
1) L→R:

FIG. 5-2.



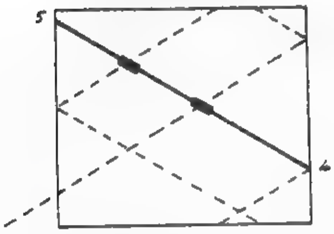
2) R→L : O.

FIG. 5-3.



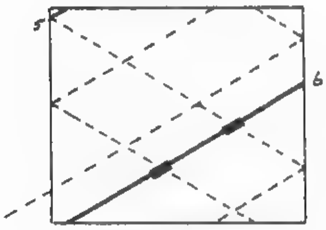
3) L→R : O.

FIG. 5-4.



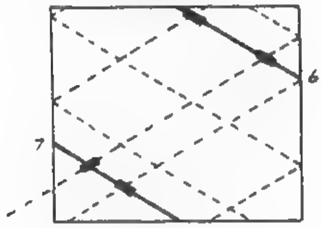
4) R→L: 2 O

FIG. 5-5.



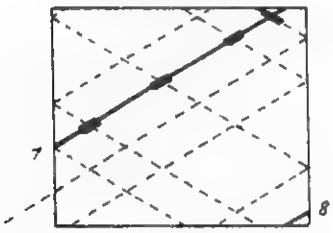
5) L→R: 2 O.

FIG. 5-6.



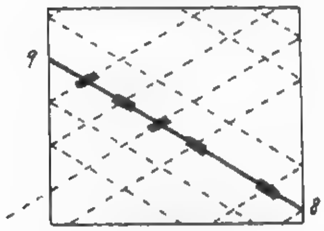
6) R→L: 3 O – U.

FIG. 5-7.



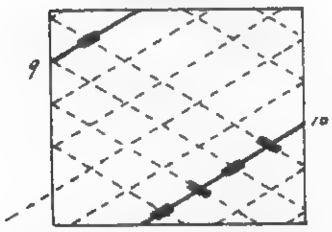
7) L→R: 3 O – U.

FIG. 5-8.



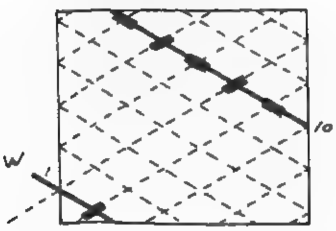
8) R→L : 2 O – U – O – U.

FIG. 5-9.



9) L→R · 2 O – U – O – U.

FIG. 5-10.



10) R→L: O – U – O – U – O – U.

1) L→R :	FREE RUN.
2) R→L :	O.
3) L→R :	O.
4) R→L :	2 O.
5) L→R :	2 O.
6) R→L :	3 O – U.
7) L→R :	3 O – U.
8) R→L :	2 O – U – O – U.
9) L→R :	2 O – U – O – U.
10) R→L :	O – U – O – U – O – U.

end over the last elastic band to the right, rotate the stick another three quarters of a turn towards you and lead the working end over the second half-cycle and feed it under the right elastic band; the result is shown in Fig. 5–3. Turn the working end to the left and follow the movement as indicated in Fig. 5–4: (working end passes over the first and third half-cycle). Now follow through with Fig. 5–5, followed by Figs. 5–6 through to 5–10 whereby the working end joins the standing end at 1.

Cut the elastic bands and remove them.

To complete the knot properly we now take the working end and follow the first half-cycle with the movements over-under-under-under-under-under and out to the right; then retract the standing end for the first crossing on the left (which was an over-crossing on the sixth half-cycle) and feed the standing end back again but now under the sixth half-cycle and out to the left.

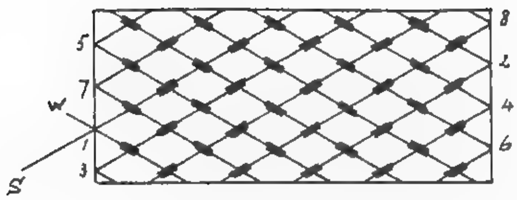
Gradually work all the slack out of the strings by transferring the knot to successively smaller-diameter objects.

An example of a longer knot is given in Fig. 6 where the construction of an 11 part/ 4 bight turk's head is illustrated. The successive half-cycle movements are given in Figs. 6–1 through to 6–8.

Finish the knot off by leading the working end along the first half-cycle under-over-under-under-under-under-under-under-under-under and out to the right. Retract the standing end for the first two crossings (which are an under-crossing on the sixth half-cycle and an over-crossing on the fourth half-cycle) and feed the standing end back again but now under the fourth half-cycle and under the sixth half-cycle and out to the left. Tighten the knot up as before.

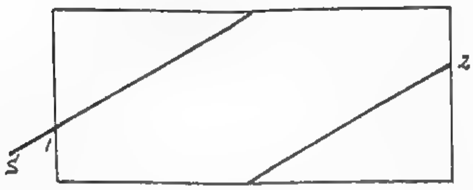
An example of a larger diameter knot is given in Fig. 7 where the construction of the 4 part/11 bight turk's head is treated.

FIG. 6.



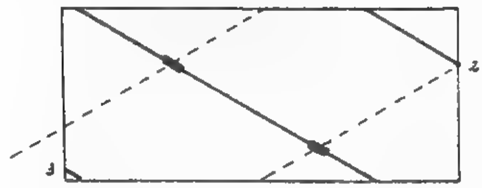
11 PART – 4 BIGHT TURK'S HEAD

FIG. 6-1.



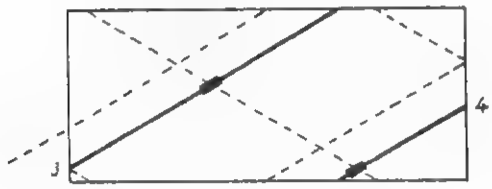
1) L→R:

FIG. 6-2.



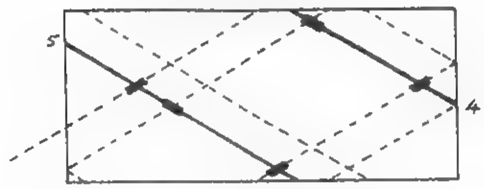
2) R→L: 2 O.

FIG. 6-3.



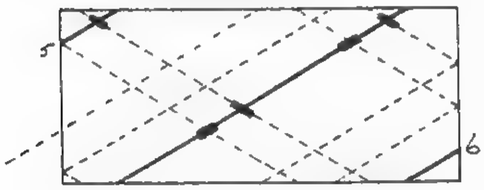
3) L→R: 2 O.

FIG. 6-4.



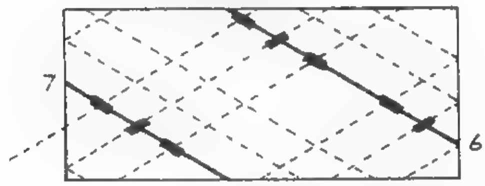
4) R→L: U-O-U-O-U.

FIG. 6-5.



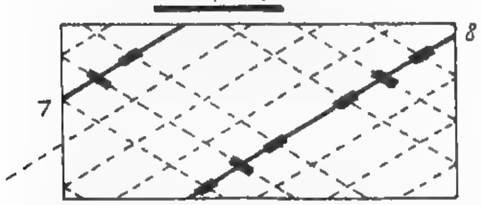
5) L→R: U-O-U-O-U.

FIG. 6-6.



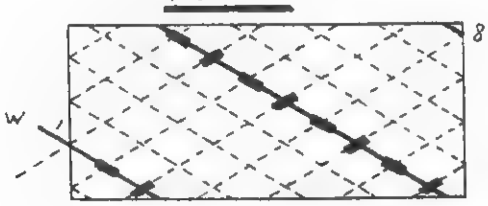
6) R→L: U-2O-U-2O-U-O.

FIG. 6-7.



7) L→R: U-2O-U-2O-U-O.

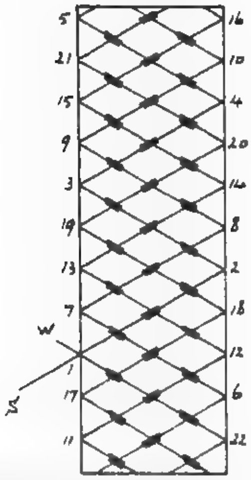
FIG. 6-8.



8) R→L: U-O-U-O-U-O-U-O-U-O.

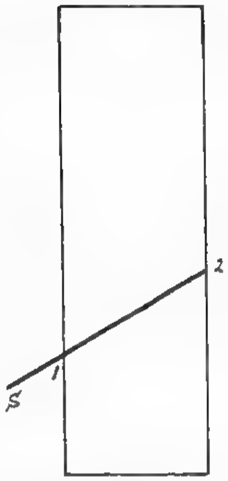
1) L→R:	FREE RUN.
2) R→L:	2 O.
3) L→R:	2 O.
4) R→L:	U-O-U-O-U
5) L→R:	U-O-U-O-U.
6) R→L:	U-2O-U-2O-U-O.
7) L→R:	U-2O-U-2O-U-O.
8) R→L:	U-O-U-O-U-O-U-O-U-O.

FIG. 7



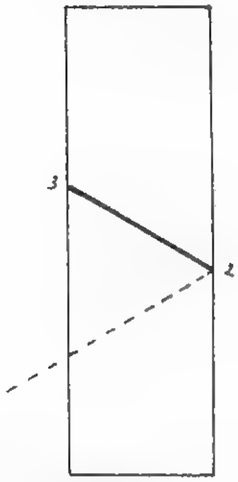
4 PART-11 BIGHT TURK'S HEAD.

FIG. 7-1



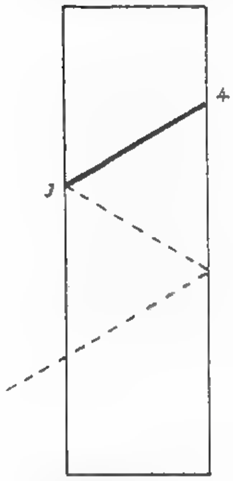
1) L→R:

FIG. 7-2.



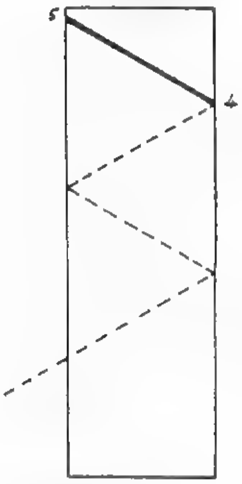
2) R→L:

FIG. 7-3.



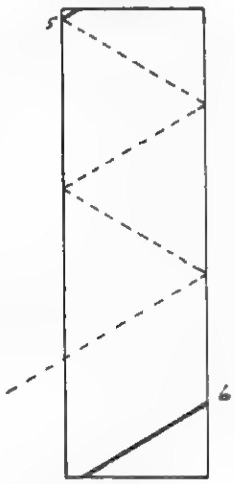
3) L→R:

FIG. 7-4.



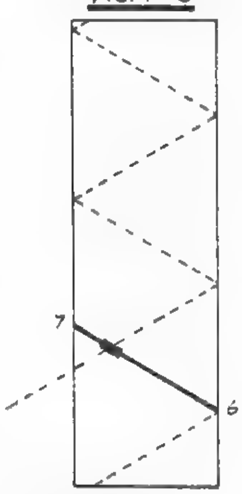
4) R→L:

FIG. 7-5.



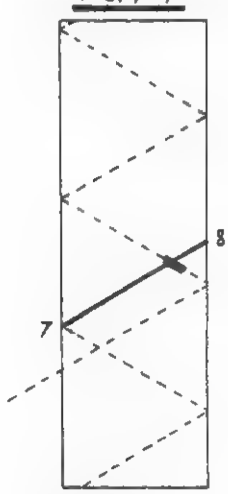
5) L→R:

FIG. 7-6.



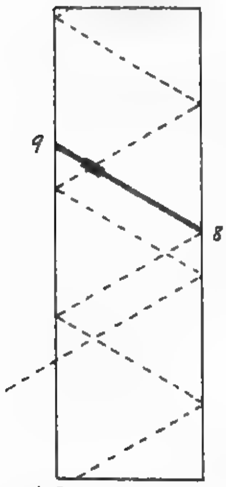
6) R→L: O.

FIG. 7-7.



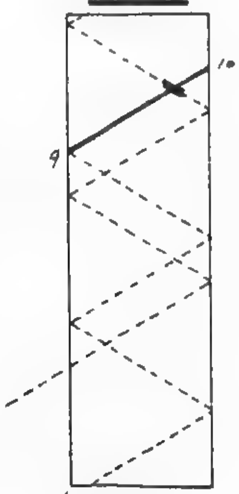
7) L→R: U.

FIG. 7-8.



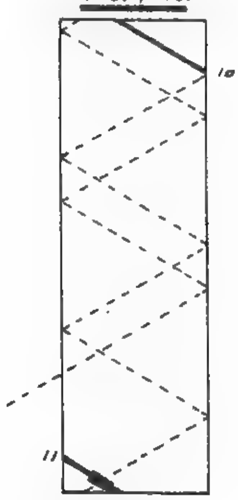
8) R→L: O.

FIG. 7-9.



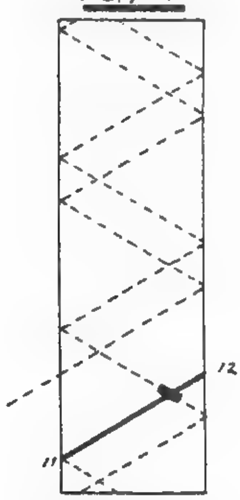
9) L→R: U.

FIG. 7-10.



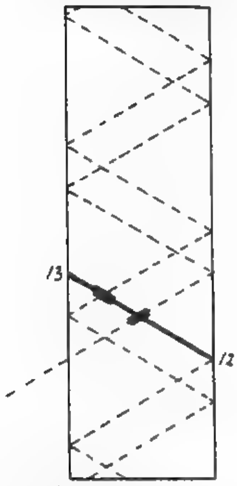
10) R→L: O

FIG. 7-11.



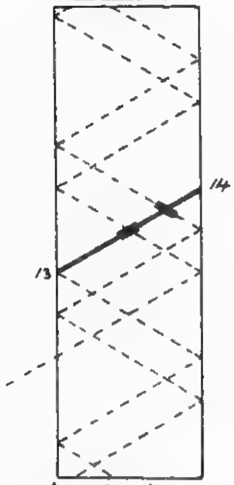
11) L→R: U.

FIG. 7-12.



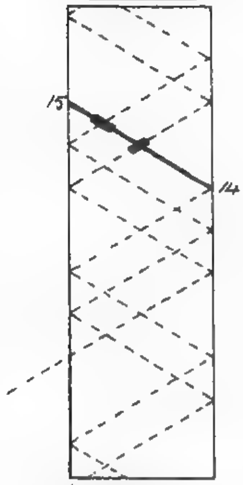
12) R→L: U-O.

FIG. 7-13.



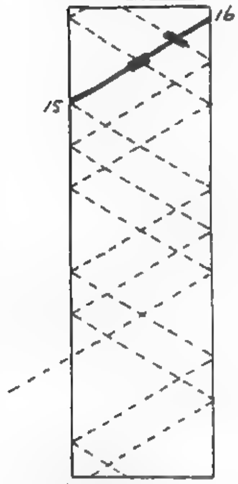
13) L→R: O-U.

FIG. 7-14.



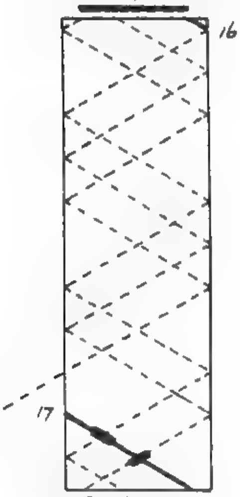
14) R→L: U-O.

FIG. 7-15.



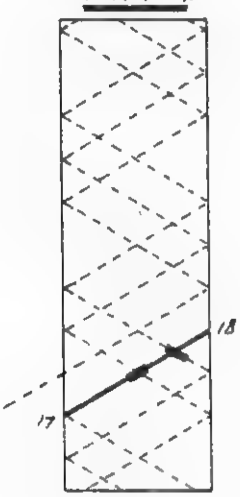
15) L→R: O-U.

FIG. 7-16



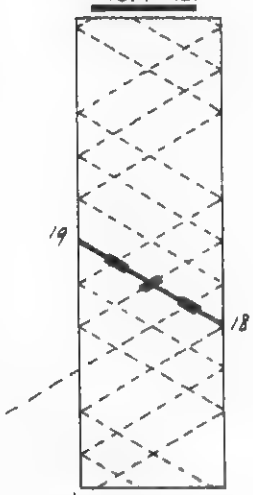
16) R→L: U-O.

FIG. 7-17.



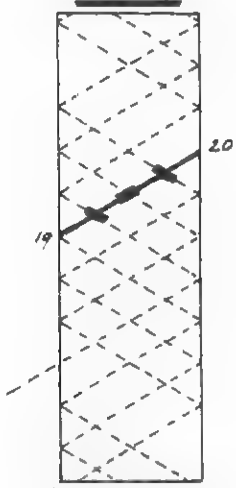
17) L→R: O-U.

FIG. 7-18.



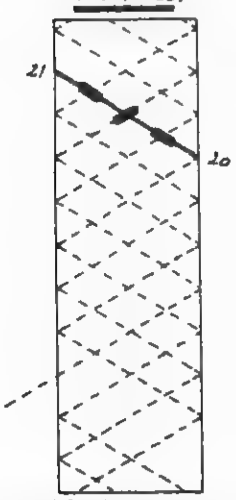
18) R→L: O-U-O.

FIG. 7-19.



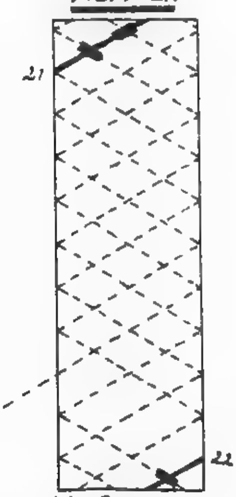
19) L→R: U-O-U.

FIG. 7-20.



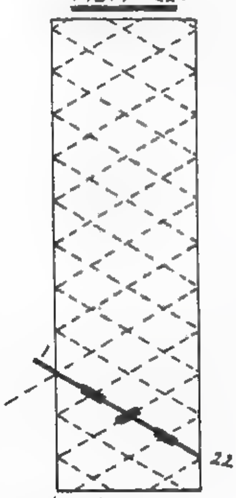
20) R→L: O-U-O.

FIG. 7-21.



21) L→R: U-O-U.

FIG. 7-22.



22) R→L: O-U-O.

1) L→R:	FREE RUN.
2) R→L:	FREE RUN.
3) L→R:	FREE RUN.
4) R→L:	FREE RUN.
5) L→R:	FREE RUN.
6) R→L:	O.
7) L→R:	U.
8) R→L:	O.
9) L→R:	U.
10) R→L:	O.
11) L→R:	U.
12) R→L:	U-O.
13) L→R:	O-U.
14) R→L:	U-O.
15) L→R:	O-U.
16) R→L:	U-O.
17) L→R:	O-U.
18) R→L:	O-U-O.
19) L→R:	U-O-U.
20) R→L:	O-U-O.
21) L→R:	U-O-U.
22) R→L:	O-U-O.

6

COLUMN-CODED REGULAR KNOTS

In this Section we give various methods for producing column-coded regular knots (see page 20 for the definition of constant column-coding).

It will be obvious how a braiding algorithm can be obtained from the grid diagram of a knot: first draw the string run and coding, then place a piece of tracing paper or other transparent medium over it and retrace the knot, half-cycle by half-cycle, starting at S, and noting down and marking the types of crossings encountered at each half-cycle. Although it is an easy method, it is very tedious, especially for large knots. Much easier and quicker is the calculation of a braiding algorithm, by one or other of the methods described below. The second one is more general, and may be found easier to use. For proofs of the formulae used see the research report [RR 1/2].

Method I

The algorithm is derived from consideration of solutions of a generating formula which relates b, p and C, which are respectively the numbers of bights and parts in the knot, and the number indicating the column where intersections (crossings) occur.

The ***generating formula*** is given by :

$$bA - pD = C, \qquad \text{with} \quad 1 \leq C \leq p - 1.$$

where $b =$ number of bights of the regular column-coded knot,
$p =$ number of parts of the regular column-coded knot, and
$C =$ column number where string intersections occur, whereby the columns are numbered from left to right for the odd numbered (1, 3, 5, ...) half-cycles and from right to left for the even (2, 4, 6, ...) half-cycles.

(N.B. the symbols $1 \leq C \leq p - 1$ mean that C can be any whole number between 1 and $p - 1$ inclusive. For example, if $p = 5$, then C is 1 or 2 or 3 or 4.)

I) The first half-cycle (from left to right) never has intersections.

II) For the $2n$th half-cycle (even-numbered, with string worked from right to left) where n is 1, 2, 3, 4, …, b, the column numbers C are the values of C in the generating formula $bA - pD = C$ such that $1 \leq C \leq (p-1)$ and where D takes on the values 0, 1, 2, 3, …, $(n-1)$ respectively and whereby for each value of D, the value of A can be 1, 2, 3, 4, ….

Example:
Say $n = 3$ then $2n = 2 \times 3 = 6$ (an even number).
Say $b = 11$ and $p = 8$; the generating formula becomes:
$11A - 8D = C, \quad 1 \leq C \leq 7 \quad (8-1=7)$.
Values for D are 0, 1, 2. $(n-1 = 3-1 = 2)$ then:
for $D = 0 \rightarrow 11A = C, \quad 1 \leq C \leq 7$. No possible C values.
for $D = 1 \rightarrow 11A - 8 = C, \quad 1 \leq C \leq 7$.
only for $A = 1$ can we find an acceptable C value : $11 - 8 = 3 = C$
for $D = 2 \rightarrow 11A - 16 = C, \quad 1 \leq C \leq 7$.
only for $A = 2$ can we find an acceptable C value : $22 - 16 = 6 = C$
Thus the intersection column numbers are 3 and 6.

III) For the $(2n+1)^{th}$ half-cycle (which is odd-numbered, with the string worked from left to right) where n is 1, 2, 3, 4, 5, …, $(b-1)$, the column numbers C are the same as those found for the half-cycle numbered $2n$. Thus the 3rd half-cycle uses the same column numbers as the 2nd; the 5th the same as the 4th, and so on. Note, however, that the column *numbers* are the same but *not* the columns since, for the odd half-cycle, columns are numbered from left to right whilst for the even half-cycle they are numbered from right to left.
We shall illustrate this method by two examples, namely the 8/11 turk's head and the 21/8 two-pass gaucho knots. Diagrams and algorithm tables for each of these appear after the working sections, on pages 32, 33, 35, 36, 37.

Example 1: *Turk's Head Knot* 8 parts/11 bights ($p/b = 8/11$).

	7	6	5	4	3	2	1	Column numbers
	↓	↓	↓	↓	↓	↓	↓	Right to Left (even half-cycles)
	\	/	\	/	\	/	\	—Column Coding
	↑	↑	↑	↑	↑	↑	↑	
Column Numbers Left to Right (odd half-cycles)	1	2	3	4	5	6	7	

Generating formula: $11A - 8D = C$, with $1 \leq C \leq 7$.

1) Left to Right : **free run.**

2) Right to Left : $n = 1$; $n - 1 = 0$; therefore $D = 0$.
The formula $11A - 8D = C, \quad 1 \leq C \leq 7$,
becomes $11A = C$; and thus there is no acceptable value for C, and therefore half-cycle 2 is also a **free run.**

3) Left to Right : the same as half-cycle 2; and therefore also half-cycle 3 is a **free run**.

4) Right to Left : $n = 2$; $n - 1 = 1$; and therefore $D = 0, 1$.
for $D = 0$: no acceptable C values (see above);
for $D = 1$: $11A - 8 = C, \quad 1 \leq C \leq 7$.
Thus only for $A = 1$ we find an acceptable C value, equal to 3. Since column 3 has an *over* coding for the right to left half-cycles, we obtain for half-cycle 4 : **o** (meaning pass the string over).

5) Left to Right : the same as half-cycle 4; and thus $C = 3$. Since column 3 has an *under* coding for the left to right half-cycles, we obtain for half-cycle 5 : **u** (meaning pass the string under).

6) Right to Left : $n = 3$; $n - 1 = 2$; and therefore $D = 0, 1, 2$.
for $D = 0$: no acceptable C values (see above);
for $D = 1$: intersection column $C = 3$ (see above);
for $D = 2$: $11A - 16 = C, \quad 1 \leq C \leq 7$.
Thus only for $A = 2$ do we find an acceptable C value, equal to 6. Since the intersection columns for the sixth half-cycle are 3 and 6; and since 3 is an *over* and 6 an *under* coding for the right to left half-cycles, we obtain: **o–u**.

7) Left to Right : the same as half-cycle 6; thus intersection columns are 3 and 6. Since 3 is an *under* and 6 an *over* coding for the left to right half-cycles, we obtain for half-cycle 7 : **u–o**.

8) Right to Left : $n = 4$; $n - 1 = 3$; therefore $D = 0, 1, 2, 3$.
for $D = 0$: no acceptable C value (see above);
for $D = 1$: intersection column $C = 3$ (see above);
for $D = 2$: intersection column $C = 6$ (see above);
for $D = 3$: $11A - 24 = C, \quad 1 \leq C \leq 7$.
Thus no acceptable C values.
Since the intersection columns for the 8th half-cycle are 3 and 6, we obtain for this half-cycle : **o–u**.

9) Left to Right : the same as half-cycle 8 and thus we obtain : **u–o**.

10) Right to Left : $n = 5$; $n - 1 = 4$; and therefore $D = 0, 1, 2, 3, 4$.
for $D = 0$: no acceptable C value (see above);
for $D = 1$: intersection column $C = 3$ (see above);
for $D = 2$: intersection column $C = 6$ (see above);
for $D = 3$: no acceptable C value (see above);
for $D = 4$: $11A - 32 = C, \quad 1 \leq C \leq 7$.
Thus only for $A = 3$ do we find an acceptable C value equal to 1.
Since the intersection columns for the 10th half-cycle are 1, 3 and 6; and since 1 is an *over*, 3 an *over* and 6 an *under* coding for the right to left half-cycles, we obtain : **2o–u**.

11) Left to Right : the same as half-cycle 10; and thus intersection columns are 1, 3 and 6.
Since 1 is an *under*, 3 an *under* and 6 an *over* coding for the left to right half-cycles, we obtain : **2u–o**.

12) Right to Left : $n = 6$; $n - 1 = 5$; therefore $D = 0, 1, 2, 3, 4, 5$.
for $D = 0, 1, 2, 3$ and 4: see above;
for $D = 5 : 11A - 40 = C, \quad 1 \leq C \leq 7$.
Thus only for $A = 4$ do we find an acceptable C value equal to 4.
Since the intersection columns for the 12th half-cycle are 1, 3, 4 and 6; and since 1 is an *over*, 3 an *over*, 4 an *under* and 6 an *under* coding, we obtain : **2o–2u.**

13) Left to Right : the same as half-cycle 12; and since 1 is an *under*, 3 an *under*, 4 an *over* and 6 an *over* coding, we obtain: **2u–2o.**

14) Right to Left : $n = 7$; $n - 1 = 6$; and therefore $D = 0, 1, 2, 3, 4, 5, 6$.
for $D = 0, 1, 2, 3, 4, 5$: see above;
for $D = 6 : 11A - 48 = C, \quad 1 \leq C \leq 7$.
Thus only for $A = 5$ do we find an acceptable C value equal to 7.
Since the intersection columns for the 14th half-cycle are 1, 3, 4, 6 and 7; and since 1 is an *over*, 3 an *over*, 4 an *under*, 6 an *under* and 7 an *over* coding, we obtain : **2o–2u–o.**

15) Left to Right : the same as half-cycle 14; and since 1 is an *under*, 3 an *under*, 4 an *over*, 6 an *over* and 7 an *under* coding, we obtain : **2u–2o–u.**

16) Right to Left : $n = 8$; $n - 1 = 7$; therefore $D = 0, 1, 2, 3, 4, 5, 6, 7$.
for $D = 0, 1, 2, 3, 4, 5, 6$: see above;
for $D = 7 : 11A - 56 = C, \quad 1 \leq C \leq 7$.
There are no acceptable values for C, and thus we obtain the same as for half-cycle 14 : **2o–2u–o.**

17) Left to Right : the same as half-cycles 16 and 15 : **2u–2o–u** .

18) Right to Left : $n = 9$; $n - 1 = 8$; therefore $D = 0, 1, 2, 3, 4, 5, 6, 7, 8$.
for $D = 0, 1, 2, 3, 4, 5, 6, 7$: see above;
for $D = 8 : 11A - 64 = C, \quad 1 \leq C \leq 7$.
Thus only for $A = 6$ do we find an acceptable C value equal to 2.
The intersection columns for the 18th half-cycle are therefore 1, 2, 3, 4, 6 and 7; and since 1 is an *over*, 2 an *under*, 3 an *over*, 4 an *under*, 6 an *under* and 7 an *over* coding, we obtain : **o–u–o–2u–o.**

19) Left to Right : the same as for half-cycle 18; thus $C = 1, 2, 3, 4, 6, 7$; and since 1 is an *under*, 2 an *over*, 3 an *under*, 4 an *over*, 6 an *over* and 7 an *under* coding, we obtain : **u–o–u–2o–u.**

20) Right to Left : $n = 10$; $n - 1 = 9$; therefore $D = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.
for $D = 0, 1, 2, 3, 4, 5, 6, 7$ and 8: see above;
for $D = 9 : 11A - 72 = C, \quad 1 \leq C \leq 7$.
Thus only for $A = 7$ do we find an acceptable C value equal to 5.
The intersection columns for the 20th half-cycle are therefore 1, 2, 3, 4, 5, 6 and 7; and since 1 is an *over*, 2 an *under*, 3 an *over*, 4 an *under*, 5 an *over*, 6 an *under* and 7 an *over* coding, we obtain : **o–u–o–u–o–u–o.**

21) Left to Right : the same as for half-cycle 20. Thus $C = 1, 2, 3, 4, 5, 6, 7$; whereby 1 is an *under*, 2 an *over*, 3 an *under*, 4 an *over*, 5 an *under*, 6 an *over* and 7 an *under* coding and we obtain : **u–o–u–o–u–o–u.**

22) Right to Left : $n = 11$; $n - 1 = 10$. Therefore $D = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$.
for $D = 0, 1, 2, 3, 4, 5, 6, 7$, 8 and 9: see above;
for $D = 10 : 11A - 80 = C$, $1 \le C \le 7$.
No acceptable values for C exist and we thus obtain the same as for half-cycle 20:
o – u – o – u – o – u – o.
We have now returned to our starting point and the knot is therefore completed. After a little practice all of the above calculations can be done easily, with very little time being required to do them. The second example, which follows the table below, gives much less explanation.

The above calculations can be tabulated as follows:

Knot: Turk's Head $p = 8$; $b = 11$; $11A - 8D = C$; $1 \le C \le 7$.

Column Coding		\	/	\	/	\	/	\	Summary
Column Number		1	2	3	4	5	6	7	
Reference Left to Right:		u	o	u	o	u	o	u	
Reference Right to Left:		o	u	o	u	o	u	o	
1	L→R								free run.
2	R→L								free run.
3	L→R								free run.
4	R→L			o					o.
5	L→R			u					u.
6	R→L			o			u		o-u.
7	L→R			u			o		u-o.
8	R→L			o			u		o-u.
9	L→R			u			o		u-o.
10	R→L	o		o			u		2o-u.
11	L→R	u		u			o		2u-o.
12	R→L	o		o	u		u		2o-2u.
13	L→R	u		u	o		o		2u-2o.
14	R→L	o		o	u		u	o	2o-2u-o.
15	L→R	u		u	o		o	u	2u-2o-u.
16	R→L	o		o	u		u	o	2o-2u-o.
17	L→R	u		u	o		o	u	2u-2o-u.
18	R→L	o	u	o	u		u	o	o-u-o-2u-o.
19	L→R	u	o	u	o		o	u	u-o-u-2o-u.
20	R→L	o	u	o	u	o	u	o	o-u-o-u-o-u-o.
21	L→R	u	o	u	o	u	o	u	u-o-u-o-u-o-u.
22	R→L	o	u	o	u	o	u	o	o-u-o-u-o-u-o.

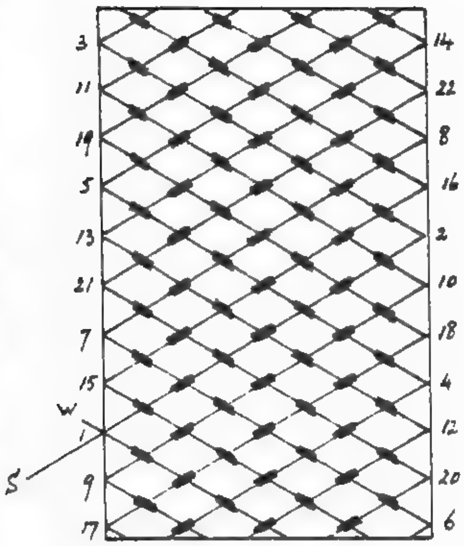


FIG. 8 TURK'S HEAD.
P/b = 8/11.

HALF.CYCLE 1,2,3,4.

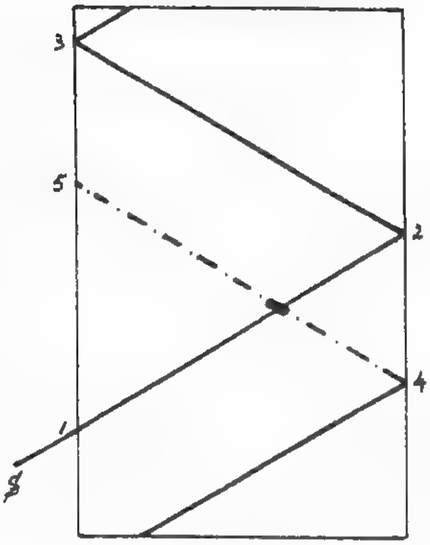


FIG. 8-1.

HALF.CYCLE 5,6,7.

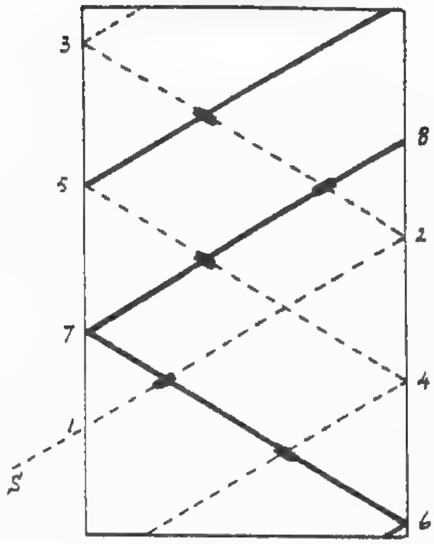


FIG. 8-2.

HALF.CYCLE 8,9,10.

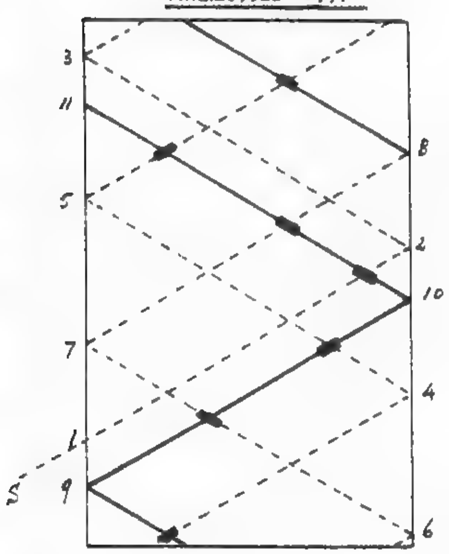


FIG. 8-3.

HALF.CYCLE 11.12.13.

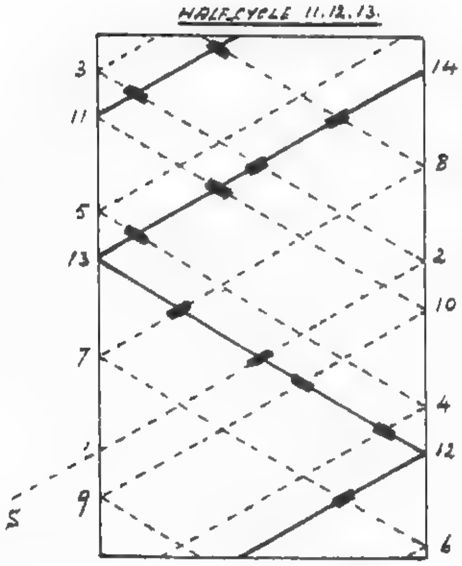


FIG. 8-4.

HALF.CYCLE 14,15,16.

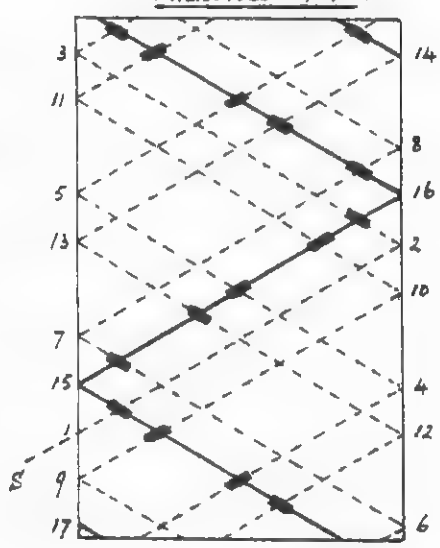
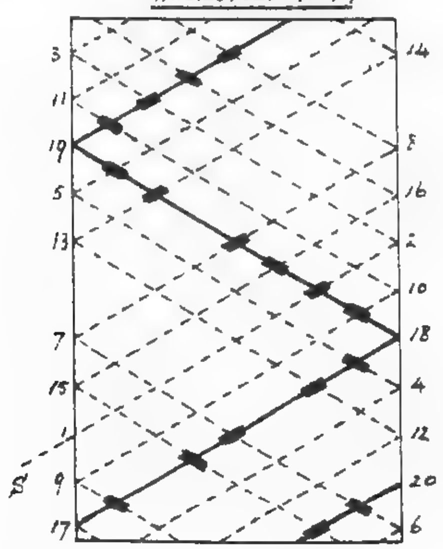
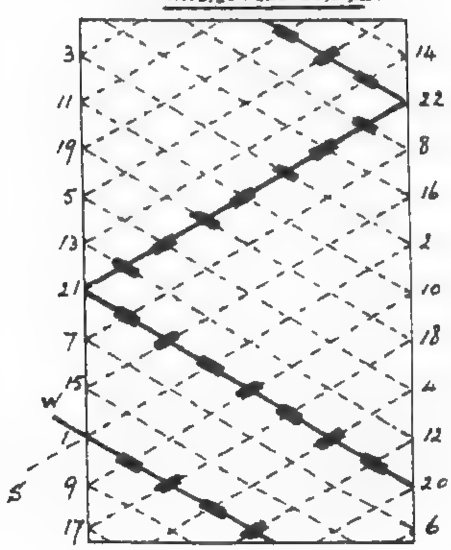


FIG 8-5

HALF.CYCLE 17.18,19.



HALF.CYCLE 20,21,22.



Example 2: *Two-pass Gaucho Knot,* $p/b = 21/8$.

(R→L)	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
Coding	/	/	\	\	/	/	\	\	/	/	\	\	/	/	\	\	/	/	\	\
(L→R)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

$p = 21$ so $p - 1 = 20$. Since $b = 8$ there is a total of $2b = 16$ half-cycles. The ***generating formula*** becomes:

$$8A - 21D = C, \quad \text{with } 1 \leq C \leq 20.$$

1) L→R : **free run.**
2) R→L : $C = 8, 16$. $(= 8, 16)$ giving two unders : **2u.**
3) L→R : $C = 8, 16$. $(= 8, 16)$ giving two unders : **2u.**
4) R→L : $C = 8, 16, 3, 11, 19$. $(=3, 8, 11, 16, 19)$ giving : **5u.**
5) L→R : $C = 8, 16, 3, 11, 19$. $(=3, 8, 11, 16, 19)$ giving : **5u.**
6) R→L : $C = 8, 16, 3, 11, 19, 6, 14$. $(= 3, 6, 8, 11, 14, 16, 19)$: **u–o–2u–o–2u.**
7) L→R : $C = 8, 16, 3, 11, 19, 6, 14$. $(= 3, 6, 8, 11, 14, 16, 19)$: **u–o–2u–o–2u.**
8) R→L : $C = 8, 16, 3, 11, 19, 6, 14, 1, 9, 17$. $(= 1, 3, 6, 8, 9, 11, 14, 16, 17, 19)$, giving :
 o–u–o–u–o–u–o–u–o–u.
9) L→R : $C = 1, 3, 6, 8, 9, 11, 14, 16, 17, 19$, giving : **o–u–o–u–o–u–o–u–o–u.**
10) R→L : $C = 8, 16, 3, 11, 19, 6, 14, 1, 9, 17, 4, 12, 20$.
 $(= 1, 3, 4, 6, 8, 9, 11, 12, 14, 16, 17, 19, 20)$, giving :
 o–2u–o–u–o–2u–o–u–o–2u.
11) L→R : $C = 1, 3, 4, 6, 8, 9, 11, 12, 14, 16, 17, 19, 20$, giving :
 o–2u–o–u–o–2u–o–u–o–2u.
12) R→L : $C = 8, 16, 3, 11, 19, 6, 14, 1, 9, 17, 4, 12, 20, 7, 15$.
 $(= 1, 3, 4, 6, 7, 8, 9, 11, 12, 14, 15, 16, 17, 19, 20)$, giving :
 o–2u–o–2u–o–2u–o–2u–o–2u.
13) L→R : $C = 1, 3, 4, 6, 7, 8, 9, 11, 12, 14, 15, 16, 17, 19, 20$, giving :
 o–2u–o–2u–o–2u–o–2u–o–2u.
14) R→L : $C = 8, 16, 3, 11, 19, 6, 14, 1, 9, 17, 4, 12, 20, 7, 15, 2, 10, 18$.
 $(= 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20)$, giving :
 2o–2u–o–2u–2o–2u–o–2u–2o–2u.
15) L→R : $C = 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20$, giving :
 2o–2u–o–2u–2o–2u–o–2u–2o–2u.
16) R→L : $C = 8, 16, 3, 11, 19, 6, 14, 1, 9, 17, 4, 12, 20, 7, 15, 2, 10, 18, 5, 13$.
 $(= 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20)$, giving :
 2o–2u–2o–2u–2o–2u–2o–2u–2o–2u.

Note that for:

2) $C = 8, 8 + 8 = 16$.
4) $C = 8, 16, 16 + 8 - 21 = 3, 3 + 8 = 11, 11 + 8 = 19$.
6) $C = 8, 16, 3, 11, 19, 19 + 8 - 21 = 6, 6 + 8 = 14$.
8) $C = 8, 16, 3, 11, 19, 6, 14, 14 + 8 - 21 = 1, 1 + 8 = 9, 9 + 8 = 17$.
 etc.
 remembering that $b = 8$ and $p = 21$ and $1 \leq C \leq 20$.

The above calculations can be tabulated as follows:

Knot: Two-pass Gaucho Knot $p = 21$, $b = 8$; $8A - 21D = C$; $1 \leq C \leq 20$.

Column Coding		/	/	\	\	/	/	\	\	/	/	\	\	/	/	\	\	/	/	\	\	Summary
Column Number		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Ref: L→R		o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	
Ref: R→L		o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	
1	L→R																					free run.
2	R→L								u								u					2u.
3	L→R								u								u					2u.
4	R→L			u					u			u					u			u		5u.
5	L→R			u					u			u					u			u		5u.
6	R→L			u			o		u			u			o		u			u		u–o–2u–o–2u.
7	L→R			u			o		u			u			o		u			u		u–o–2u–o–2u.
8	R→L	o		u			o		u	o		u			o		u	o		u		o–u–o–u–o–u–o–u–o–u.
9	L→R	o		u			o		u	o		u			o		u	o		u		o–u–o–u–o–u–o–u–o–u.
10	R→L	o		u	u		o		u	o		u	u		o		u	o		u	u	o–2u–o–u–o–2u–o–u–o–2u.
11	L→R	o		u	u		o		u	o		u	u		o		u	o		u	u	o–2u–o–u–o–2u–o–u–o–2u.
12	R→L	o		u	u		o	u	u	o		u	u		o	u	u	o		u	u	o–2u–o–2u–o–2u–o–2u–o–2u.
13	L→R	o		u	u		o	u	u	o		u	u		o	u	u	o		u	u	o–2u–o–2u–o–2u–o–2u–o–2u.
14	R→L	o	o	u	u		o	u	u	o	o	u	u		o	u	u	o	o	u	u	2o–2u–o–2u–2o–2u–o–2u–2o–2u.
15	L→R	o	o	u	u		o	u	u	o	o	u	u		o	u	u	o	o	u	u	2o–2u–o–2u–2o–2u–o–2u–2o–2u.
16	R→L	o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	2o–2u–2o–2u–2o–2u–2o–2u–2o–2u.

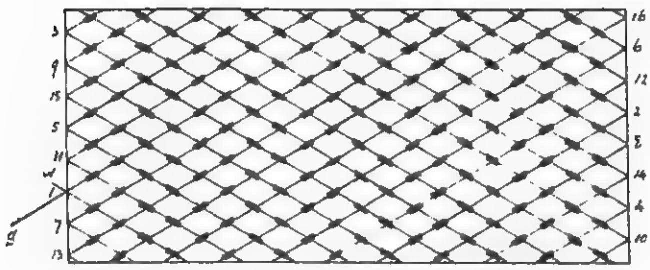


FIG 9 TWO-PASS GAUCHO KNOT P/b = 2 1/8

HALF-CYCLE 1&2.

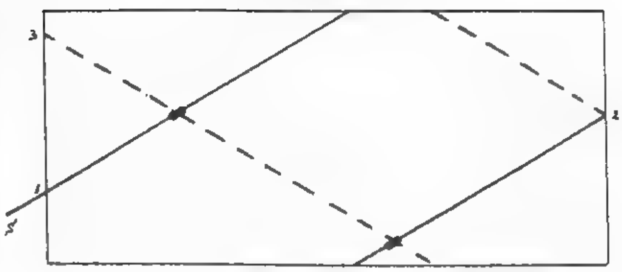


FIG 9-1

HALF-CYCLE 3

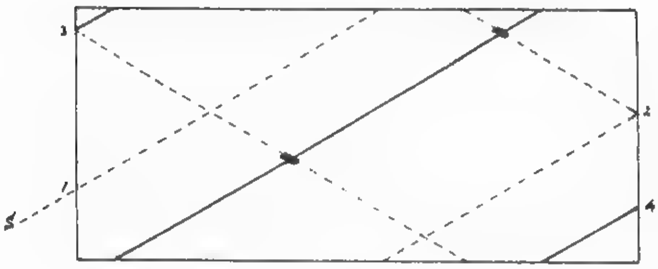


FIG 9-2

HALF-CYCLE 4

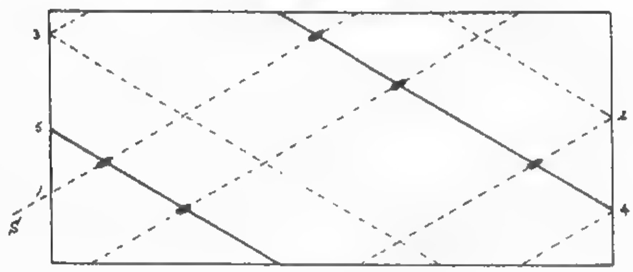


FIG. 9-3

HALF-CYCLE 5

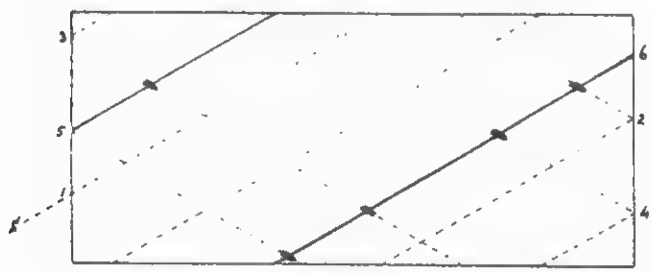


FIG. 9-4

HALF-CYCLE 6

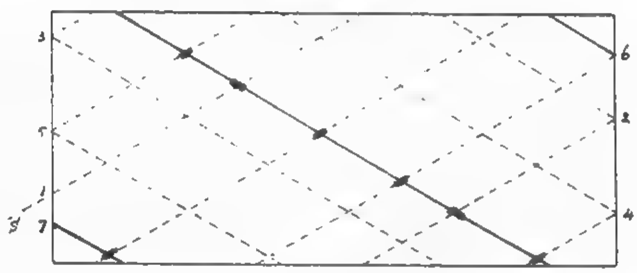


FIG. 9-5

HALF-CYCLE 7

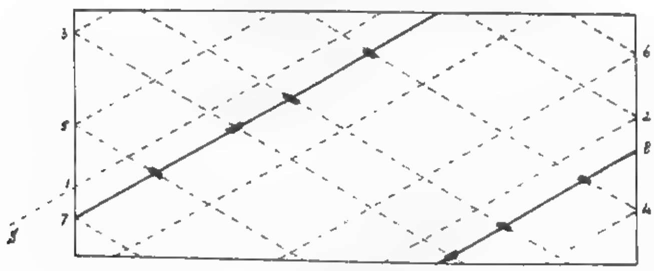


FIG 9-6.

HALF-CYCLE 8

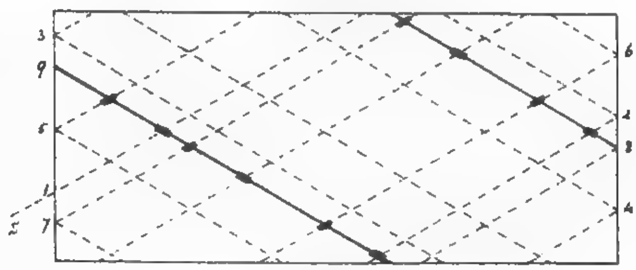


FIG 9-7.

HALF-CYCLE 9

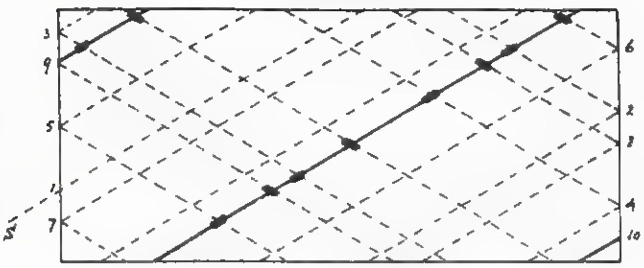


FIG 9-8

HALF-CYCLE 10.

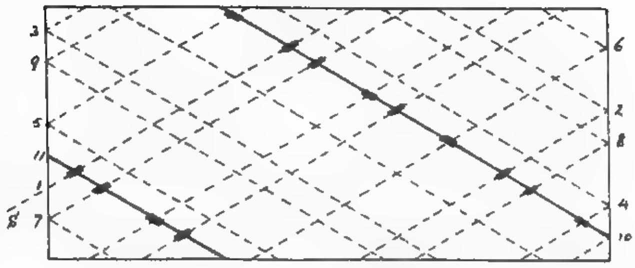


FIG 9-9

HALF-CYCLE 11

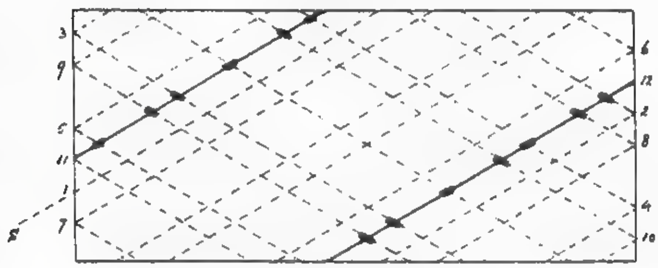


FIG. 9-10

HALF-CYCLE 12

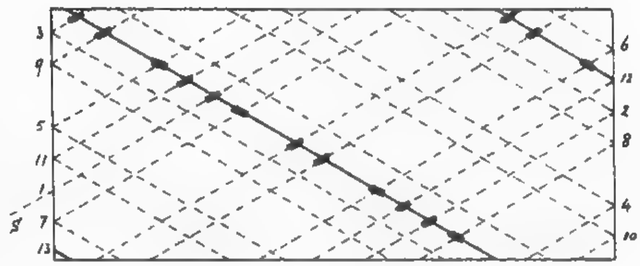


FIG 9-11

HALF-CYCLE 13

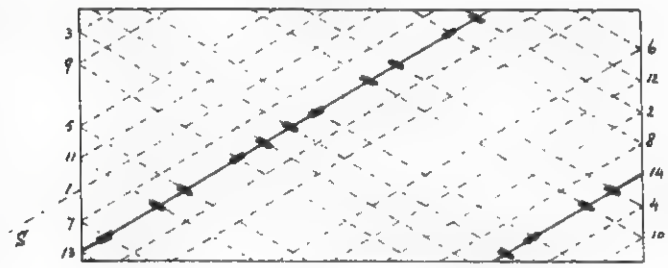


FIG 9-12

HALF-CYCLE 14

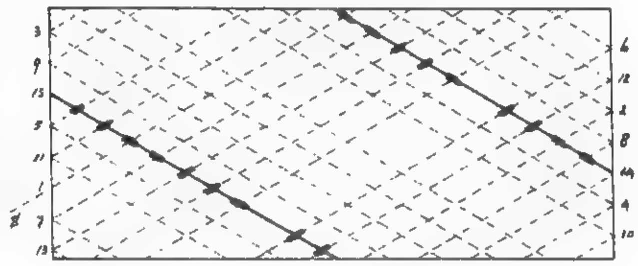


FIG. 9-13

HALF-CYCLE 15

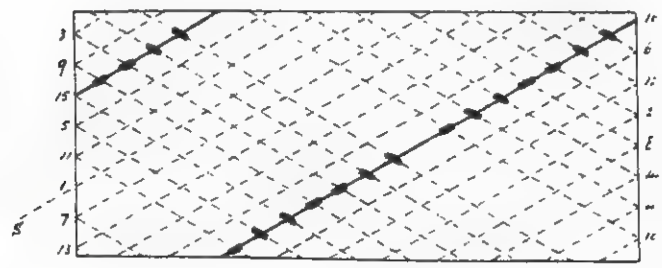


FIG 9-14

HALF-CYCLE 16

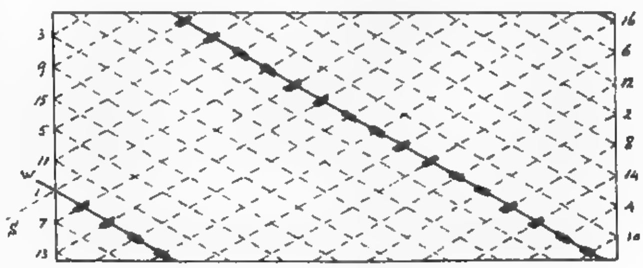


FIG 9-15

Method II

The second method for generating a braiding algorithm involves a different formula, which this time relates p and b (parts and bights) to a variable I which is a bight index number. The meaning of I is best understood by considering the diagram on the facing page (Fig. 10), which represents a continued layout of the string run as the braiding progresses.

I_0 is the starting point (standing end); the first bight formed on the right-hand side is also indicated by I_0; I_1 indicates the first bight formed on the left-hand side, then another I_1 occurs at the second bight formed on the right-hand side; and so on for I_2, I_2, I_3, I_3, ..., etc. The diagram shows the knot 'unrolled', as it were, from the cylinder; no details of the interweaving are shown at this stage.

The ***generating formula*** used is:

$$pK^* - bL^* = I^*.$$

We first transform this formula into:

$$(p - nb)K - bL = I, \qquad \text{with } 0 \le I \le b,$$

where p = number of parts of the regular knot,
b = number of bights of the regular knot,
I = bight index number,
n = a whole number such that $1 \le (p - nb) \le b - 1$,
K = a whole number 0, 1, 2, 3, ..., b.
L = a whole number 0, 1, 2, 3, ..., $(p - nb)$.

Draw the diagram representing the $2b$ half-cycles starting at the bottom left and ending at the top left as shown in Fig. 10.

The I_0 value is equal to zero ($K = 0$, $L = 0$) and the I_b value is equal to b and to 0 (cyclic relationship $I_b = I_0$; $K = b$, $L = p - nb - 1$ and $L = p - nb$).

The first half-cycle is always a free run and the column numbers where intersections occur for the *odd* half-cycle are the same as those for the immediately preceding *even* half-cycle. The columns are again numbered as before : from the starting point of the half-cycle concerned to its end point.

Column numbers where intersections occur for the *even* half-cycle I_R to I_L are:

$$C = (b - I_m) + n^*b, \qquad \text{with } 1 \le C \le p - 1,$$

where m = 0, 1, 2, 3, ... R,
n^* = 0, 1, 2, 3, ...

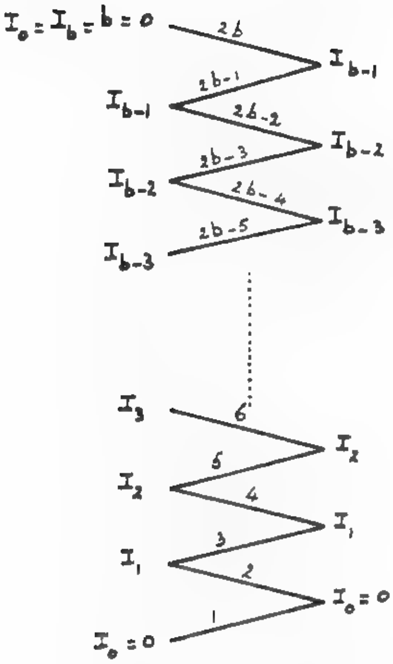


FIG. 10.

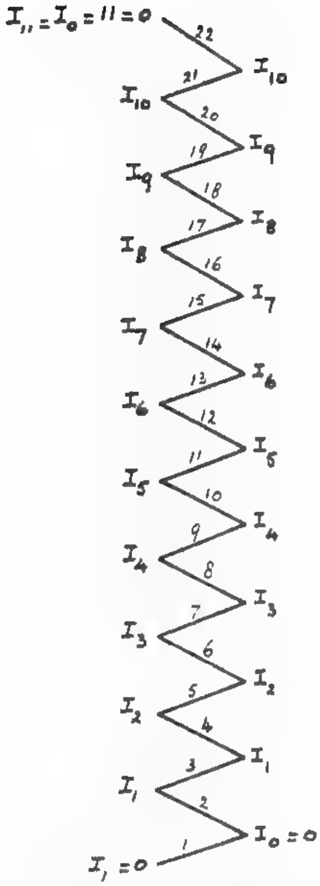


FIG. 11-1.

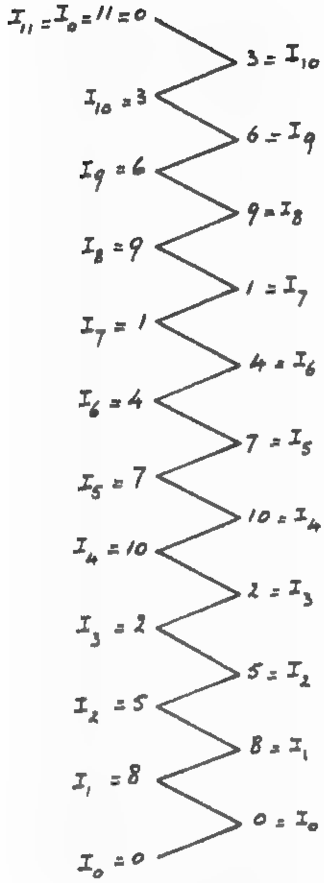


FIG. 11-2.

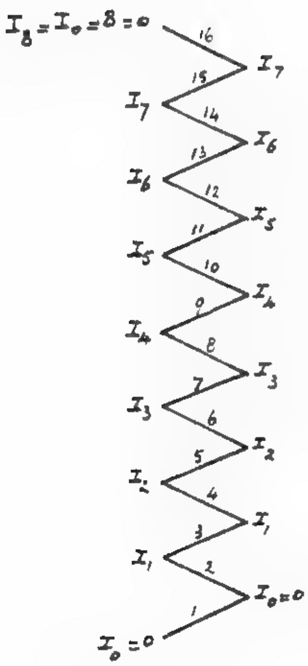


FIG. 12-1

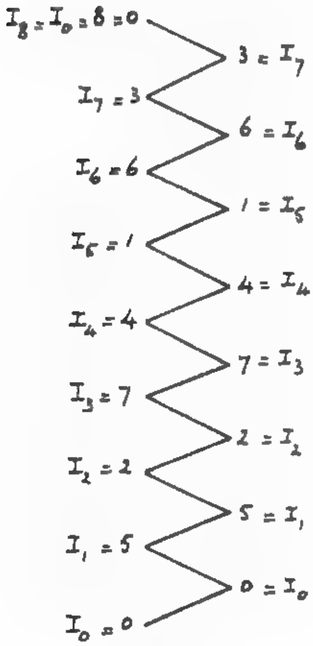


FIG. 12-2.

Example 1: *Turk's Head Knot,* 8 parts/11 bights (same as on page 29; see also the diagrams of page 33).

The ***generating formula*** becomes $8K^* - 11L^* = I^*$ and this transforms into:

$$8K - 11L = I, \qquad 0 \leq I \leq 11.$$

(n = 0 and therefore no change in transformation).

	7	6	5	4	3	2	1	Column numbers from
	↓	↓	↓	↓	↓	↓	↓	right to left (even half-cycles)
Column coding →	\	/	\	/	\	/	\	
	↑	↑	↑	↑	↑	↑	↑	
Column numbers from left to right (odd half-cycles)	1	2	3	4	5	6	7	

Since $b = 11$ draw the following diagram with $2b = 22$ half-cycles and mark the bottom left- and right-hand bight points 0 ($I_0 = 0$), ($K = 0$, $L = 0$ in the generating formula $8K - 11L = I$). See Figs. 11–1 and 11–2 on page 39.

The next bight points up, left and right, are equal to $I_1 = 8$, ($K = 1$, $L = 0$).

The next bight points up, left and right, are equal to $I_2 = 5$, ($K = 2$, $L = 1$).

The next bight points up, left and right, are equal to $I_3 = 2$, ($K = 3$, $L = 2$) ;
(or $I_3 = 5 + 8 - 11 = 2$).

The next bight points up, left and right, are equal to $I_4 = 10$, ($K = 4$, $L = 2$);
(or $I_4 = 2 + 8 = 10$).

The next bight points up, left and right, are equal to $I_5 = 7$, ($K = 5$, $L = 3$);
(or $I_5 = 10 + 8 - 11 = 7$).

The next bight points up, left and right, are equal to $I_6 = 4$, ($K = 6$, $L = 4$);
(or $I_6 = 7 + 8 - 11 = 4$).

The next bight points up, left and right, are equal to $I_7 = 1$, ($K = 7$, $L = 5$) ;
(or $I_7 = 4 + 8 - 11 = 1$).

The next bight points up, left and right, are equal to $I_8 = 9$, ($K = 8$, $L = 5$) ;
(or $I_8 = 1 + 8 = 9$).

The next bight points up, left and right, are equal to $I_9 = 6$, ($K = 9$, $L = 6$) ;
(or $I_9 = 9 + 8 - 11 = 6$).

The next bight points up, left and right, are equal to $I_{10} = 3$, ($K = 10$, $L = 7$);
(or $I_{10} = 6 + 8 - 11 = 3$).

The next bight point up, left is equal to $I_{11} = 11 = 0$, ($K = 11$, $L = 7$ and $L = 8$);
(or $I_{11} = 3 + 8 = 11 = 3 + 8 - 11 = 0$).

The first half-cycle runs from left bight index number 0 to right bight index number 0. No intersections (always a **free run**).

The second half-cycle runs from right bight index number $I_R = I_0 = 0$ to the left bight index number $I_L = I_1 = 8$. Since $R = 0$, the m value can only be equal to 0 and thus we would obtain $C = (11 - 0) + 11n^*$; but since $1 \leq C \leq 7$ we have no acceptable C values and as a consequence again we have a **free run**.

The third half-cycle has the same C values as the second half-cycle and since the second half-cycle did not have any acceptable C values, also the third half-cycle is a **free run**.

The fourth half-cycle runs from the right bight index number $I_R = I_1 = 8$ to the left bight index number $I_L = I_2 = 5$. Since $R = 1$, the m values are 0, 1.
For $m = 0$ we obtain $C = (11 - I_0) + 11n^* = (11 - 0) + 11n^* = 11 + 11n^*$. But $1 \leq C \leq 7$, therefore no acceptable C values.
For $m = 1$ we obtain $C = (11 - I_1) + 11n^* = (11 - 8) + 11n^* = 3 + 11n^*$. Since $1 \leq C \leq 7$, the only acceptable C value is equal to 3, $(n^* = 0)$.
Thus overall for the fourth half-cycle we found $\boldsymbol{C = 3}$.

The fifth half-cycle has the same C values as the fourth half-cycle and thus $\boldsymbol{C = 3}$.

The sixth half-cycle runs from right bight index number $I_R = I_2 = 5$ to the left bight index number $I_L = I_3 = 2$. Since $R = 2$, the m values are 0, 1, 2.
for $m = 0$ see above; no C values.
for $m = 1$ see above; $C = 3$.
for $m = 2$ we obtain $C = (11 - I_2) + 11n^* = (11 - 5) + 11n^* = 6 + 11n^*$.
Since $1 \leq C \leq 7$, the only acceptable C value is equal to 6. Thus overall for the sixth half-cycle we found $\boldsymbol{C = 3, 6}$.

The seventh half-cycle has the same C values as the sixth half-cycle and thus $\boldsymbol{C = 3, 6}$.

The eighth half-cycle runs from right bight index number $I_R = I_3 = 2$ to the left bight index number $I_L = I_4 = 10$. Since $R = 3$ the m values are 0, 1, 2, 3.
for $m = 0$ see above; no C values.
for $m = 1$ see above; $C = 3$.
for $m = 2$ see above; $C = 6$.
for $m = 3$ we obtain $C = (11 - I_3) + 11n^* = (11 - 2) + 11n^* = 9 + 11n^*$.
But $1 \leq C \leq 7$ and therefore no acceptable C values. Thus overall we obtain for the eighth half-cycle : $\boldsymbol{C = 3, 6}$.

The ninth half-cycle has the same C values as the eighth half-cycle and thus $\boldsymbol{C = 3, 6}$.

The tenth half-cycle runs from right bight index number $I_R = I_4 = 10$ to the left bight index number $I_L = I_5 = 7$. Since $R = 4$ the m values are 0, 1, 2, 3, 4.
for $m = 0$ see above; no C values.
for $m = 1$ see above; $C = 3$.
for $m = 2$ see above; $C = 6$.
for $m = 3$ see above; no C values.
for $m = 4$ we obtain $C = (11 - I_4) + 11n^* = (11 - 10) + 11n^* = 1 + 11n^*$.
Since $1 \leq C \leq 7$, the only acceptable C value is equal to 1. Thus overall for the tenth half-cycle we found $\boldsymbol{C = 1, 3, 6}$.

In a similar fashion the remainder are as follows:

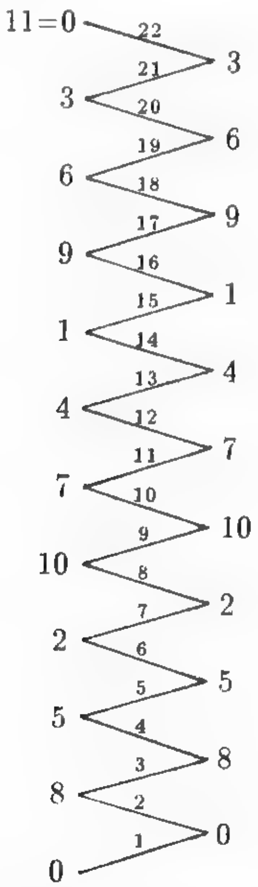
11th half-cycle : as the 10th half-cycle, therefore $\boldsymbol{C = 1, 3, 6}$.
12th half-cycle : $I_R = I_5 = 7$; $m = 0, 1, 2, 3, 4, 5$; $\boldsymbol{C = 1, 3, 4, 6}$.
13th half-cycle : as the 12th half-cycle; $\boldsymbol{C = 1, 3, 4, 6}$.
14th half-cycle : $I_R = I_6 = 4$; $m = 0, 1, 2, 3, 4, 5, 6$; $\boldsymbol{C = 1, 3, 4, 6, 7}$.
15th half-cycle : as the 14th half-cycle; $\boldsymbol{C = 1, 3, 4, 6, 7}$.
16th half-cycle : $I_R = I_7 = 1$; $m = 0, 1, 2, 3, 4, 5, 6, 7$; $\boldsymbol{C = 1, 3, 4, 6, 7}$.
17th half-cycle : as the 16th half-cycle; $\boldsymbol{C = 1, 3, 4, 6, 7}$.
18th half-cycle : $I_R = I_8 = 9$; $m = 0, 1, 2, 3, 4, 5, 6, 7, 8$; $\boldsymbol{C = 1, 2, 3, 4, 6, 7}$.
19th half-cycle : as the 18th half-cycle; $\boldsymbol{C = 1, 2, 3, 4, 6, 7}$.
20th half-cycle : $I_R = I_9 = 6$; $m = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$; $\boldsymbol{C = 1, 2, 3, 4, 5, 6, 7}$.
21st half-cycle : as the 20th half-cycle; $\boldsymbol{C = 1, 2, 3, 4, 5, 6, 7}$.
22nd half-cycle : $I_R = I_{10} = 3$; $m = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$;
$\boldsymbol{C = 1, 2, 3, 4, 5, 6, 7}$.

Now we can read off the braiding algorithm for each half-cycle from the column numbers and their association with the column coding.

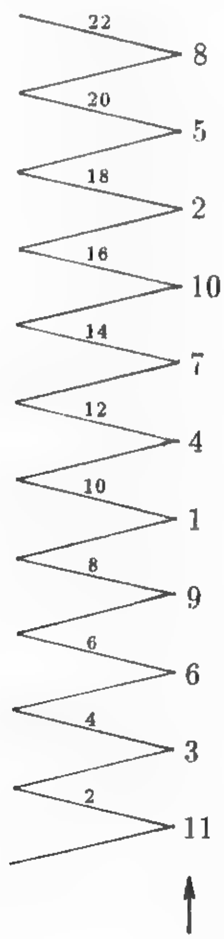
Let us summarise the above given calculation method:

$$p = 8; \qquad b = 11; \qquad 8K^* - 11L^* = I^* \rightarrow 8K - 11L = I.$$

Draw the bight index diagram as on the left.



bight index numbers I.



first intersection column number with the first half-cycle on the indicated even half-cycles.

Calculate the *first* intersection column number with the first half-cycle on each *even* half-cycle (half-cycle from lower right to upper left) by subtracting the bight index from the number of bights, and again graph this in the diagram as on the right.

The intersection column numbers for an even half-cycle are all those of the even half-cycles passed *plus* (the *first* intersection column number + $11n^*$) of the half-cycle under consideration, provided $1 \leq C \leq 7$.

The odd half-cycle has the same intersection column numbers as the preceding even half-cycle.

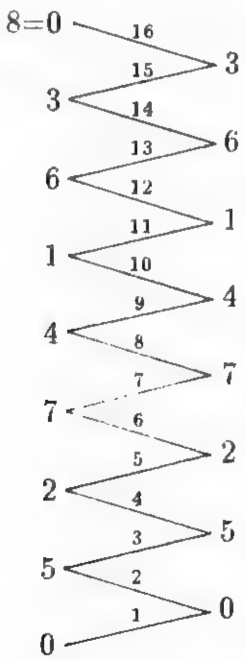
Example 2: *Two-pass Gaucho Knot* $p/b = 21/8$ (same as on page 34; see also the diagrams on page 36,37).

The ***generating formula*** is $21K^* - 8L^* = I^*$. This is transformed into:

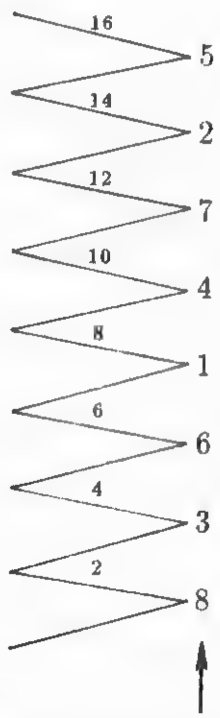
$$5K - 8L = I \qquad (n = 2), \qquad 0 \leq I \leq 8.$$

(R→L)	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
Coding	/	/	\	\	/	/	\	\	/	/	\	\	/	/	\	\	/	/	\	\
(L→R)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

See Figs. 12–1 and 12–2 for bight index diagrams (page 39).



bight index numbers I.



first intersection column number with the first half-cycle on the indicated even half-cycles.

1) L→R : **free run.**

2) R→L : $I_R = I_0 = 0$; $m = 0$; $\boldsymbol{C} = \mathbf{8, 16}$.

3) L→R : $\boldsymbol{C} = \mathbf{8, 16}$.

4) R→L : $I_R = I_1 = 5$; $m = 0, 1$; $\boldsymbol{C} = \mathbf{3, 8, 11, 16, 19}$.

5) L→R : $\boldsymbol{C} = \mathbf{3, 8, 11, 16, 19}$.

6) R→L : $I_R = I_2 = 2$; $m = 0, 1, 2$; $\boldsymbol{C} = \mathbf{3, 6, 8, 11, 14, 16, 19}$.

7) L→R : $\boldsymbol{C} = \mathbf{3, 6, 8, 11, 14, 16, 19}$.

8) R→L : $I_R = I_3 = 7$; $m = 0, 1, 2, 3$; $\boldsymbol{C} = \mathbf{1, 3, 6, 8, 9, 11, 14, 16, 17, 19}$.

9) L→R : $\boldsymbol{C} = \mathbf{1, 3, 6, 8, 9, 11, 14, 16, 17, 19}$.

10) R→L : $I_R = I_4 = 4$; $m = 0, 1, 2, 3, 4$; $\boldsymbol{C} = \mathbf{1, 3, 4, 6, 8, 9, 11, 12, 14, 16, 17, 19, 20}$.

11) L→R : $\boldsymbol{C} = \mathbf{1, 3, 4, 6, 8, 9, 11, 12, 14, 16, 17, 19, 20}$.

12) R→L : $I_R = I_5 = 1$; $m = 0, 1, 2, 3, 4, 5$; $\boldsymbol{C} = \mathbf{1, 3, 4, 6, 7, 8, 9, 11, 12, 14, 15, 16, 17, 19, 20}$.

13) L→R : $\boldsymbol{C} = \mathbf{1, 3, 4, 6, 7, 8, 9, 11, 12, 14, 15, 16, 17, 19, 20}$.

14) R→L : $I_R = I_6 = 6$; $m = 0, 1, 2, 3, 4, 5, 6$; $\boldsymbol{C} = \mathbf{1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20}$.

15) L→R : $\boldsymbol{C} = \mathbf{1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20}$.

16) R→L : $I_R = I_7 = 3$; $m = 0, 1, 2, 3, 4, 5, 6, 7$; $\boldsymbol{C} = \mathbf{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}$.

Now we can read off the braiding algorithm for each half-cycle from the column numbers and their associated coding (c.f. page 35).

Exercises on the use of these methods are given on pages 73 to 78.

7

ROW-CODED REGULAR KNOTS

In this section we treat the class of row-coded regular knots (see page 20 for the definition of constant row-coding). We introduce the generating formula and the braiding algorithm, referring to a 7/5 knot example. Then we give a fully worked example for a 17/6 perfect two-pass herringbone knot.

Figure 13–1 is the graphical representation of the string run of a 7 part—5 bight regular knot. Through the string intersections we can draw horizontal lines: the rows are where string intersections occur (Fig. 13–2). The horizontal line which runs through the starting bight position is numbered 0 and its bight index is I_0. The rows are now numbered as in Fig. 13–3. When we have gone completely around the knot we return again to our starting row which was already numbered 0, but is also equal to row number $2b$ where b is the number of bights of the regular knot: (in our case $2b = 2 \times 5 = 10$). The next row would be 11 but is also 1, the next would be 12 but is also 2 etc. The row numbers are cyclic in $2b$; in our case $2 \times 5 = 10$. The bight positions on the *left*-hand side are numbered as in Fig. 13–4 and are called the left-hand bight index numbers with $I_0 = 0 =$ row 0. Where the first half-cycle reaches the right-hand knot side and forms the first right-hand bight, this position is the bight index number 0 on the *right*-hand side. Again from here the rest of the bight positions on the right-hand side are numbered as in Fig. 13–5 and are the right-hand bight index numbers. We see the bight index numbers are cyclic in b (in our case $b = 5$) and thus the bight index $I_0 = I_b = 0 = b$ (mod 5).

The row number of the right-hand bight index I_0 is equal to p (in our case 7) and therefore to find the row numbers belonging to the right-hand bight positions we have to multiply the respective right-hand bight index number by two and add p.

The row number belonging to right-hand bight index I_R is equal to $2I_R + p$. To find the basic row number we have to take into account the cyclic nature of row numbers, and thus we subtract $2b$ from $(2I_R + p)$ as many times as will leave a remainder smaller than $2b$ but greater than or equal to zero. We indicate this remainder, which becomes the basic row number, by:

$$|2I_R + p|_{2b}$$

This is called the *modulus* notation. To illustrate, suppose we take $I = 3$, $p = 7$ and $b = 5$; then

$$\begin{aligned} |2I_R + p|_{2b} &= |2 \times 3 + 7|_{10} \\ &= |13|_{10} \quad \text{(say "13 modulus 10")} \\ &= 3 \end{aligned}$$

since 3 is the remainder when 13 is divided by 10.

In our example for a $p/b = 7/5$ regular knot we therefore obtain (see Fig. 13–6):

Left-hand Side	*Right-hand Side*
Start of 1st half-cycle; $I_0 = 0$; Row $= 0$	End of 1st half-cycle; Start of 2nd half-cycle; $I_0 = 0$; Row $= 7$
End of 2nd half-cycle; Start of 3rd half cycle; $I_1 = 2$; Row $= 4$	End of 3rd half-cycle; Start of 4th half-cycle; $I_1 = 2$; Row $= \|2 \times 2 + 7\|_{10} = 1$
End of 4th half-cycle; Start of 5th half-cycle; $I_2 = 4$; Row $= 8$	End of 5th half-cycle; Start of 6th half-cycle; $I_2 = 4$, Row $= \|2 \times 4 + 7\|_{10} = 5$
End of 6th half-cycle; Start of 7th half-cycle; $I_3 = 1$; Row $= 2$	End of 7th half-cycle; Start of 8th half-cycle; $I_3 = 1$; Row $= \|2 \times 1 + 7\|_{10} = 9$
End of 8th half-cycle; Start of 9th half-cycle; $I_4 = 3$; Row $= 6$	End of 9th half-cycle; Start of 10th half-cycle; $I_4 = 3$; Row $= \|2 \times 3 + 7\|_{10} = 3$
End of 10th half-cycle; $I_5 = I_0 = 5 = 0$; Row $= 0 = 10$	

The way to calculate left-hand row numbers is explained on page 48.

General Procedure:

The bight index number ***generating formula*** is in fact the formula given on page 38 :

$$(p - nb)K - bL = I, \quad \text{with } 0 \le I \le b,$$

where
- p = number of parts of the regular knot,
- b = number of bights of the regular knot,
- I = bight index number,
- n = a whole number such that $1 \le p - nb \le b - 1$,
- K = a whole number $0, 1, 2, 3, \ldots, b$,
- L = a whole number $0, 1, 2, 3, \ldots, (p - nb)$.

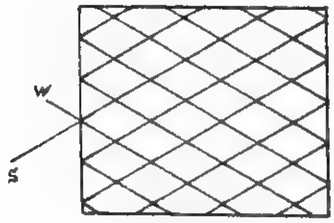


FIG. 13-1.

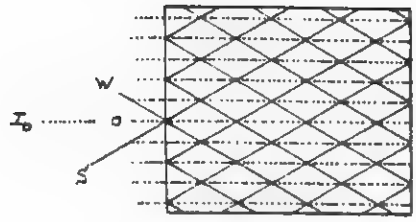


FIG. 13-2.

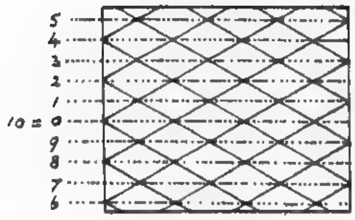


FIG. 13-3.

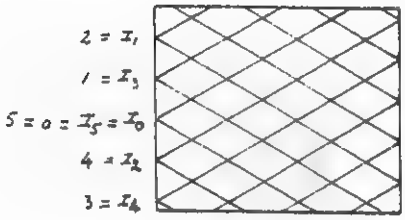


FIG 13-4.

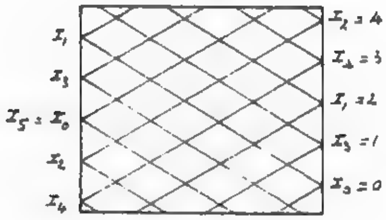


FIG. 13-5.

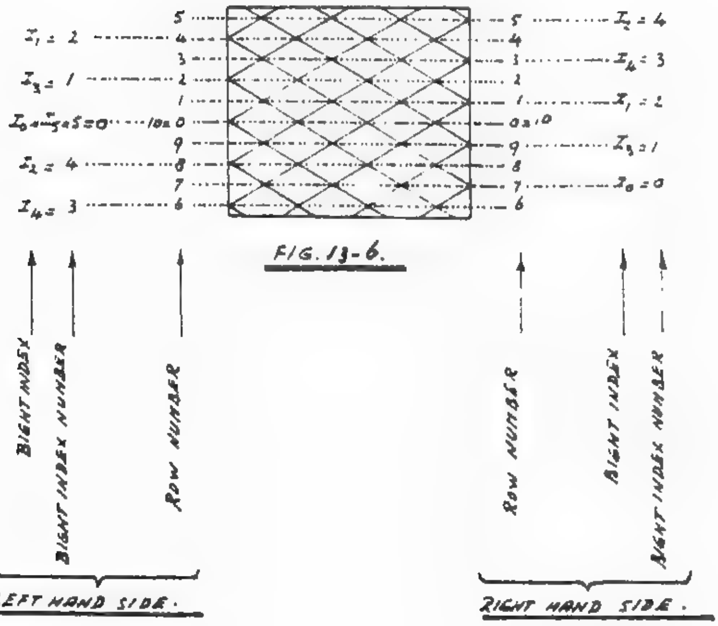


FIG. 13-6.

Draw the $2b$ half-cycles and use the bight index number generating formula to calculate the respective bight index numbers I. See Fig. 14, page 53.

To find the braiding algorithm for half-cycle I_t to I_s we have to find the sequence of *row numbers* where this half-cycle intersects the previously laid-down ones.

First calculate the sequence of the column numbers C then calculate the row numbers:

1) when t is on the left-hand side : Row $= |2I_t + C|_{2b}$
2) when t is on the right-hand side : Row $= |2I_t + p + C|_{2b}$

The calculation procedures for C are as given on page 28 or 38.

Example: *Perfect 2 Pass Herringbone Knot*, $p/b = 17/6$ (see pages 52, 53 for diagrams and table, Figures 15–1 and 15–2).

$p = 17$
$b = 6$ thus $2b = 12$

Row Coding		Row Numbers
\	⟵	11
\	⟵	10
/	⟵	9
/	⟵	8
\	⟵	7
\	⟵	6
/	⟵	5
/	⟵	4
\	⟵	3
\	⟵	2
/	⟵	1
/	⟵	0

The bight index number ***generating formula*** becomes:

$17K^* - 6L^* = I^*$, This transforms into:

$5K - 6L = I \quad (n = 2)$, with $0 \le I \le 6$, and $1 \le C \le 16$.

1) Half-cycle 1; L→R: **free run.**

2) Half-cycle 2; R→L: $C = 6, 12$.

$$\text{Row} = |2 \times 0 + 17 + 6|_{12} = |23|_{12} = 11$$
$$\text{Row} = |2 \times 0 + 17 + 12|_{12} = |29|_{12} = 5$$

Intersecting row numbers are 11, 5 → **o–u**.

3) Half-cycle 3; L→R: as in 2); thus $C = 6, 12$.

$$\begin{array}{lclclcl} \text{Row} & = & |2\times 5+6|_{12} & = & |16|_{12} & = & 4 \\ \text{Row} & = & |2\times 5+12|_{12} & = & |22|_{12} & = & 10 \end{array}$$

Intersecting row numbers are 4, 10 → **o–u.**

4) Half-cycle 4; R→L: $C = 1, 6, 7, 12, 13$.

$$\begin{array}{lclclcr} \text{Row} & = & |2\times 5+17+1|_{12} & = & |28|_{12} & = & 4 \\ \text{Row} & = & |2\times 5+17+6|_{12} & = & |33|_{12} & = & 9 \\ \text{Row} & = & |2\times 5+17+7|_{12} & = & |34|_{12} & = & 10 \\ \text{Row} & = & |2\times 5+17+12|_{12} & = & |39|_{12} & = & 3 \\ \text{Row} & = & |2\times 5+17+13|_{12} & = & |40|_{12} & = & 4 \end{array}$$

Intersecting row numbers are 4, 9, 10, 3, 4 → **2u–2o–u.**

5) Half-cycle 5; L→R: C as in 4); thus $C = 1, 6, 7, 12, 13$.

$$\begin{array}{lclclcl} \text{Row} & = & |2\times 4+1|_{12} & = & |9|_{12} & = & 9 \\ \text{Row} & = & |2\times 4+6|_{12} & = & |14|_{12} & = & 2 \\ \text{Row} & = & |2\times 4+7|_{12} & = & |15|_{12} & = & 3 \\ \text{Row} & = & |2\times 4+12|_{12} & = & |20|_{12} & = & 8 \\ \text{Row} & = & |2\times 4+13|_{12} & = & |21|_{12} & = & 9 \end{array}$$

Intersecting row numbers are 9, 2, 3, 8, 9 → **o–2u–2o.**

6) Half-cycle 6; R→L: $C = 1, 2, 6, 7, 8, 12, 13, 14$.

$$\begin{array}{lclclcl} \text{Row} & = & |2\times 4+17+1|_{12} & = & |26|_{12} & = & 2 \\ \text{Row} & = & |2\times 4+17+2|_{12} & = & |27|_{12} & = & 3 \\ \text{Row} & = & |2\times 4+17+6|_{12} & = & |31|_{12} & = & 7 \\ \text{Row} & = & |2\times 4+17+7|_{12} & = & |32|_{12} & = & 8 \\ \text{Row} & = & |2\times 4+17+8|_{12} & = & |33|_{12} & = & 9 \\ \text{Row} & = & |2\times 4+17+12|_{12} & = & |37|_{12} & = & 1 \\ \text{Row} & = & |2\times 4+17+13|_{12} & = & |38|_{12} & = & 2 \\ \text{Row} & = & |2\times 4+17+14|_{12} & = & |39|_{12} & = & 3 \end{array}$$

Intersecting row numbers are 2, 3, 7, 8, 9, 1, 2, 3 → **3o–3u–2o.**

7) Half-cycle 7; L→R: C as in 6); thus $C = 1, 2, 6, 7, 8, 12, 13, 14$.

$$\begin{array}{lclclcl} \text{Row} & = & |2\times 3+1|_{12} & = & |7|_{12} & = & 7 \\ \text{Row} & = & |2\times 3+2|_{12} & = & |8|_{12} & = & 8 \\ \text{Row} & = & |2\times 3+6|_{12} & = & |0|_{12} & = & 0 \\ \text{Row} & = & |2\times 3+7|_{12} & = & |13|_{12} & = & 1 \\ \text{Row} & = & |2\times 3+8|_{12} & = & |14|_{12} & = & 2 \\ \text{Row} & = & |2\times 3+12|_{12} & = & |18|_{12} & = & 6 \\ \text{Row} & = & |2\times 3+13|_{12} & = & |19|_{12} & = & 7 \\ \text{Row} & = & |2\times 3+14|_{12} & = & |20|_{12} & = & 8 \end{array}$$

Intersecting row numbers are 7, 8, 0, 1, 2, 6, 7, 8 → **u–3o–3u–o.**

8) Half-cycle 8; R→L: $C = 1, 2, 3, 6, 7, 8, 9, 12, 13, 14, 15$.

$$
\begin{array}{lclclcl}
\text{Row} & = & |2\times 3+17+1|_{12} & = & |24|_{12} & = & 0 \\
\text{Row} & = & |2\times 3+17+2|_{12} & = & |25|_{12} & = & 1 \\
\text{Row} & = & |2\times 3+17+3|_{12} & = & |26|_{12} & = & 2 \\
\text{Row} & = & |2\times 3+17+6|_{12} & = & |29|_{12} & = & 5 \\
\text{Row} & = & |2\times 3+17+7|_{12} & = & |30|_{12} & = & 6 \\
\text{Row} & = & |2\times 3+17+8|_{12} & = & |31|_{12} & = & 7 \\
\text{Row} & = & |2\times 3+17+9|_{12} & = & |32|_{12} & = & 8 \\
\text{Row} & = & |2\times 3+17+12|_{12} & = & |35|_{12} & = & 11 \\
\text{Row} & = & |2\times 3+17+13|_{12} & = & |36|_{12} & = & 0 \\
\text{Row} & = & |2\times 3+17+14|_{12} & = & |37|_{12} & = & 1 \\
\text{Row} & = & |2\times 3+17+15|_{12} & = & |38|_{12} & = & 2
\end{array}
$$

Intersecting row numbers are 0, 1, 2, 5, 6, 7, 8, 11, 0, 1, 2
→ **2u–o–u–2o–u–o–2u–o.**

9) Half-cycle 9; L→R: C as in 8); thus $C = 1, 2, 3, 6, 7, 8, 9, 12, 13, 14, 15$.

$$
\begin{array}{lclclcl}
\text{Row} & = & |2\times 2+1|_{12} & = & |5|_{12} & = & 5 \\
\text{Row} & = & |2\times 2+2|_{12} & = & |6|_{12} & = & 6 \\
\text{Row} & = & |2\times 2+3|_{12} & = & |7|_{12} & = & 7 \\
\text{Row} & = & |2\times 2+6|_{12} & = & |10|_{12} & = & 10 \\
\text{Row} & = & |2\times 2+7|_{12} & = & |11|_{12} & = & 11 \\
\text{Row} & = & |2\times 2+8|_{12} & = & |12|_{12} & = & 0 \\
\text{Row} & = & |2\times 2+9|_{12} & = & |13|_{12} & = & 1 \\
\text{Row} & = & |2\times 2+12|_{12} & = & |16|_{12} & = & 4 \\
\text{Row} & = & |2\times 2+13|_{12} & = & |17|_{12} & = & 5 \\
\text{Row} & = & |2\times 2+14|_{12} & = & |18|_{12} & = & 6 \\
\text{Row} & = & |2\times 2+15|_{12} & = & |19|_{12} & = & 7
\end{array}
$$

Intersecting row numbers are 5, 6, 7, 10, 11, 0, 1, 4, 5, 6, 7 → **o–4u–4o–2u.**

10) Half-cycle 10; R→L: $C = 1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16$.

$$
\begin{array}{lclclcl}
\text{Row} & = & |2\times 2+17+1|_{12} & = & |22|_{12} & = & 10 \\
\text{Row} & = & |2\times 2+17+2|_{12} & = & |23|_{12} & = & 11 \\
\text{Row} & = & |2\times 2+17+3|_{12} & = & |24|_{12} & = & 0 \\
\text{Row} & = & |2\times 2+17+4|_{12} & = & |25|_{12} & = & 1 \\
\text{Row} & = & |2\times 2+17+6|_{12} & = & |27|_{12} & = & 3 \\
\text{Row} & = & |2\times 2+17+7|_{12} & = & |28|_{12} & = & 4 \\
\text{Row} & = & |2\times 2+17+8|_{12} & = & |29|_{12} & = & 5 \\
\text{Row} & = & |2\times 2+17+9|_{12} & = & |30|_{12} & = & 6 \\
\text{Row} & = & |2\times 2+17+10|_{12} & = & |31|_{12} & = & 7 \\
\text{Row} & = & |2\times 2+17+12|_{12} & = & |33|_{12} & = & 9 \\
\text{Row} & = & |2\times 2+17+13|_{12} & = & |34|_{12} & = & 10 \\
\text{Row} & = & |2\times 2+17+14|_{12} & = & |35|_{12} & = & 11 \\
\text{Row} & = & |2\times 2+17+15|_{12} & = & |36|_{12} & = & 0 \\
\text{Row} & = & |2\times 2+17+16|_{12} & = & |37|_{12} & = & 1
\end{array}
$$

Intersecting row numbers are 10, 11, 0, 1, 3, 4, 5, 6, 7, 9, 10, 11, 0, 1
→ **2o–2u–o–2u–2o–u–2o–2u.**

11) Half-cycle 11; L→R: C as in 10); thus $C = 1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 13, 14, 15, 16$.

$$\begin{array}{lclclcr}
\text{Row} & = & |2\times 1+1|_{12} & = & |3|_{12} & = & 3\\
\text{Row} & = & |2\times 1+2|_{12} & = & |4|_{12} & = & 4\\
\text{Row} & = & |2\times 1+3|_{12} & = & |5|_{12} & = & 5\\
\text{Row} & = & |2\times 1+4|_{12} & = & |6|_{12} & = & 6\\
\text{Row} & = & |2\times 1+6|_{12} & = & |8|_{12} & = & 8\\
\text{Row} & = & |2\times 1+7|_{12} & = & |9|_{12} & = & 9\\
\text{Row} & = & |2\times 1+8|_{12} & = & |10|_{12} & = & 10\\
\text{Row} & = & |2\times 1+9|_{12} & = & |11|_{12} & = & 11\\
\text{Row} & = & |2\times 1+10|_{12} & = & |12|_{12} & = & 0\\
\text{Row} & = & |2\times 1+12|_{12} & = & |14|_{12} & = & 2\\
\text{Row} & = & |2\times 1+13|_{12} & = & |15|_{12} & = & 3\\
\text{Row} & = & |2\times 1+14|_{12} & = & |16|_{12} & = & 4\\
\text{Row} & = & |2\times 1+15|_{12} & = & |17|_{12} & = & 5\\
\text{Row} & = & |2\times 1+16|_{12} & = & |18|_{12} & = & 6
\end{array}$$

Intersecting row numbers are 3, 4, 5, 6, 8, 9, 10, 11, 0, 2, 3, 4, 5, 6
→ **u–2o–u–2o–2u–o–2u–2o–u**.

12) Half-cycle 12; R→L: $C = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16$.

$$\begin{array}{lclclcr}
\text{Row} & = & |2\times 1+17+1|_{12} & = & |20|_{12} & = & 8\\
\text{Row} & = & |2\times 1+17+2|_{12} & = & |21|_{12} & = & 9\\
\text{Row} & = & |2\times 1+17+3|_{12} & = & |22|_{12} & = & 10\\
\text{Row} & = & |2\times 1+17+4|_{12} & = & |23|_{12} & = & 11\\
\text{Row} & = & |2\times 1+17+5|_{12} & = & |24|_{12} & = & 0\\
\text{Row} & = & |2\times 1+17+6|_{12} & = & |25|_{12} & = & 1\\
\text{Row} & = & |2\times 1+17+7|_{12} & = & |26|_{12} & = & 2\\
\text{Row} & = & |2\times 1+17+8|_{12} & = & |27|_{12} & = & 3\\
\text{Row} & = & |2\times 1+17+9|_{12} & = & |28|_{12} & = & 4\\
\text{Row} & = & |2\times 1+17+10|_{12} & = & |29|_{12} & = & 5\\
\text{Row} & = & |2\times 1+17+11|_{12} & = & |30|_{12} & = & 6\\
\text{Row} & = & |2\times 1+17+12|_{12} & = & |31|_{12} & = & 7\\
\text{Row} & = & |2\times 1+17+13|_{12} & = & |32|_{12} & = & 8\\
\text{Row} & = & |2\times 1+17+14|_{12} & = & |33|_{12} & = & 9\\
\text{Row} & = & |2\times 1+17+15|_{12} & = & |34|_{12} & = & 10\\
\text{Row} & = & |2\times 1+17+16|_{12} & = & |35|_{12} & = & 11
\end{array}$$

Intersecting row numbers are 8, 9, 10, 11, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
→ **2u–2o–2u–2o–2u–2o–2u–2o**.

The above calculations can be tabulated as follows:

Knot: Perfect 2 Pass Herringbone Knot; $p = 17,\ b = 6;\quad 5K - 6L = I,\quad 0 \le I \le 6,\quad 1 \le C \le 16.$

RowCoding		/	/	\	\	/	/	\	\	/	/	\	\	/	/	\	\	/	/	\	\	/	/	\	\	/	/	\	Summary
Row Number		0	1	2	3	4	5	6	7	8	9	10	11	0	1	2	3	4	5	6	7	8	9	10	11	0	1	2	
Ref: L→R:		o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	
Ref: R→L:		u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	
1	L→R																												free run.
2	R→L												o						u										o–u.
3	L→R					o						u																	o–u.
4	R→L					u					u	o					o	u											2u–2o–u.
5	L→R										o					u	u					o	o						o–2u–2o.
6	R→L			o	o				o	u	u				u	o	o												3o–3u–2o.
7	L→R								u	o				o	o	u				u	u	o							u–3o–3u–o.
8	R→L	u	u	o			u	o	o	u			o	u	u	o													2u–o–u–2o–u–o–2u–o.
9	L→R						o	u	u			u	u	o	o			o	o	u	u								o–4u–4o–2u.
10	R→L											o	o	u	u		o	u	u	o	o		u	o	o	u	u		2o–2u–o–2u–2o–u–2o–2u.
11	L→R				u	o	o	u		o	o	u	u	o		u	u	o	o	u									u–2o–u–2o–2u–o–2u–2o–u.
12	R→L									u	u	o	o	u	u	o	o	u	u	o	o	u	u	o	o				2u–2o–2u–2o–2u–2o–2u–2o.

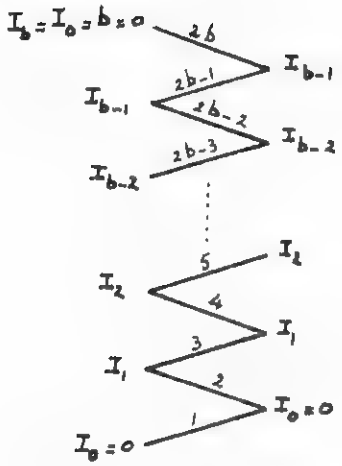


FIG. 14.

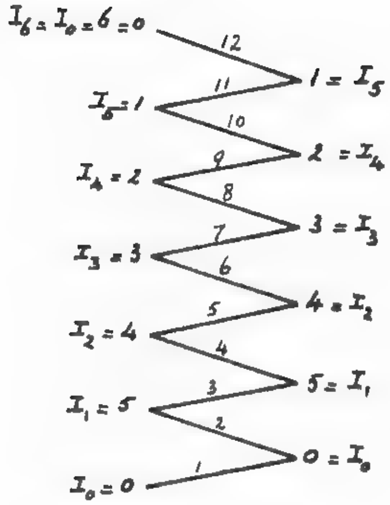
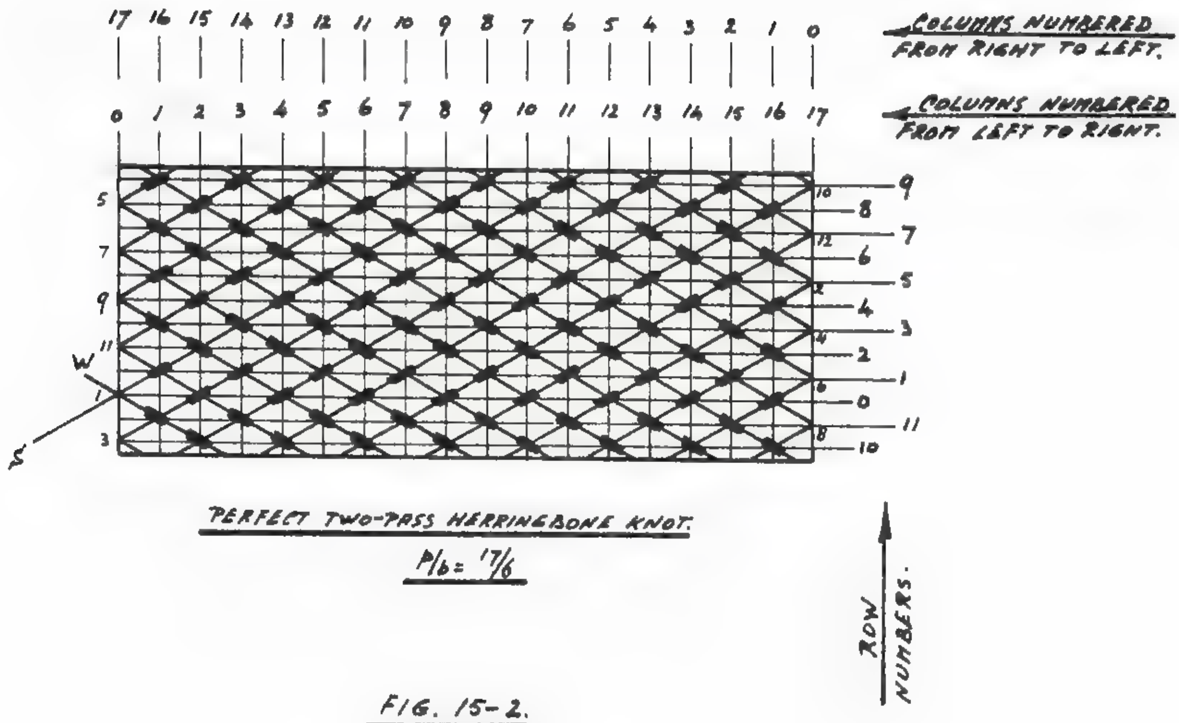


FIG. 15-1.



PERFECT TWO-PASS HERRINGBONE KNOT.

$P/b = 17/6$

FIG. 15-2.

8

REGULAR KNOTS WHICH ARE NEITHER COLUMN NOR ROW CODED

In this section we give two methods for dealing with knots which have neither constant column-coding nor constant row-coding.

The first method uses one of the generating formulae discussed under method I or method II from Section 6 to calculate the intersection column numbers for each consecutive half-cycle. Then the braiding algorithm is derived by following the half-cycle under consideration from its starting point to its end point and noting down the type of intersection (under or over) encountered for each consecutive intersection column number. The second method is more complicated, but it is suitable for generalisation and computer application.

Method I

We first draw the grid diagram of the knot in question. Then we calculate the intersection column numbers for each half-cycle and read off from the grid diagram the braiding algorithm for each of the half-cycles. The following example illustrates this procedure.

Example: *Slow Helix Knot Type II,* $p/b = 17/10$ (see Figure 16 on next page).

Intersection column number ***generating formula***:

$$10A - 17D = C, \quad \text{with } 1 \leq C \leq 16.$$

Compute the C values for each half-cycle, as described on page 28. Then use these in conjunction with the string run and coding shown on the grid diagram. A comment on how to do this is given for the 2nd half-cycle, in the table following Fig. 16.

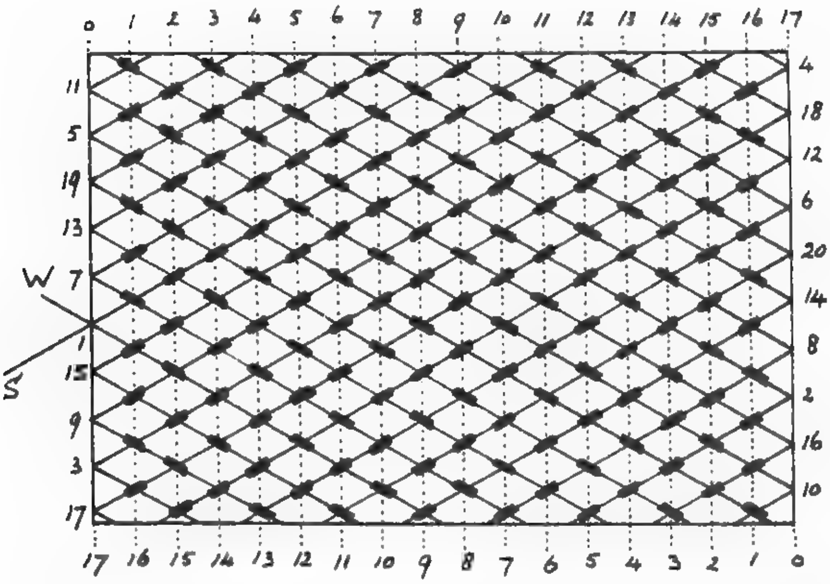


Figure 16. *Slow Helix Knot Type II* $p/b = 17/10$

1) L→R : **free run.**
2) R→L : $C = 10 \rightarrow$ **u.**
   (observe the string running from 2 on the R-edge to 3 on the L-edge of Figure 16; there is an *under* mark at column 10).
3) L→R : $C = 10 \rightarrow$ **u.**
4) R→L : $C = 3, 10, 13 \rightarrow$ **3u.**
5) L→R : $C = 3, 10, 13 \rightarrow$ **u–o–u.**
6) R→L : $C = 3, 6, 10, 13, 16 \rightarrow$ **2o–u–2o.**
7) L→R : $C = 3, 6, 10, 13, 16 \rightarrow$ **u–2o–2u.**
8) R→L : $C = 3, 6, 9, 10, 13, 16 \rightarrow$ **o–2u–3o.**
9) L→R : $C = 3, 6, 9, 10, 13, 16 \rightarrow$ **o–u–3o–u.**
10) R→L : $C = 2, 3, 6, 9, 10, 12, 13, 16 \rightarrow$ **o–3u–2o–2u.**
11) L→R : $C = 2, 3, 6, 9, 10, 12, 13, 16 \rightarrow$ **2o–u–o–u–3o.**
12) R→L : $C = 2, 3, 5, 6, 9, 10, 12, 13, 15, 16 \rightarrow$ **o–3u–o–u–o–3u.**
13) L→R : $C = 2, 3, 5, 6, 9, 10, 12, 13, 15, 16 \rightarrow$ **2o–u–o–2u–2o–u–o.**
14) R→L : $C = 2, 3, 5, 6, 8, 9, 10, 12, 13, 15, 16 \rightarrow$ **3u–o–u–o–5u.**
15) L→R : $C = 2, 3, 5, 6, 8, 9, 10, 12, 13, 15, 16 \rightarrow$ **o–u–2o–2u–2o–u–2o.**
16) R→L : $C = 1, 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16$
   $\rightarrow$ **2u–o–u–2o–2u–o–u–o–u–o.**
17) L→R : $C = 1, 2, 3, 5, 6, 8, 9, 10, 11, 12, 13, 15, 16$
   $\rightarrow$ **o–2u–2o–u–3o–2u–o–u.**
18) R→L : $C = 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16$
   $\rightarrow$ **2u–o–u–o–u–o–u–o 2u–o–u–2o.**
19) L→R : $C = 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16$
   $\rightarrow$ **o–u–3o–u–3o–2u–3o–u.**
20) R→L : $C = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16$
   $\rightarrow$ **u–o–2u–o–u–o–2u–o–u–o–2u–o–u.**

Note that this knot must have a number of bights that is a multiple of 5, as otherwise the coding is not cyclic.

Method II

Although this method is very simple, it is better suited to computer application. The great advantage of the method lies in the fact that we do not have to draw a separate diagram for each different size of the same knot type.

In general the coding of a knot can be built up out of repeating blocks, and the simplest of the non-column non-row coded knots are built up with three repeating blocks, in the following manner:

1) *sub-block I* : that builds the left-hand side of the knot;
2) *main block* : that builds the central main part of the knot;
3) *sub-block II* : that builds the right-hand side of the knot.

An example of such a knot is given on page 61 (later we treat the general case, using several repeating blocks).

The coding that belongs to a crossing between two strings in the knot is equated to an equivalent coding crossing in one of the building blocks; and the braiding algorithm is built up out of these equivalent coding crossings.

Sub-block I consists of α_I columns and $2\beta_I$ rows and builds those columns of the knot that fulfil $1 \leq C_L \leq N_L$ wherein C_L is the column number of the knot as numbered from left to right.

The main block consists of α columns and 2β rows and builds those columns of the knot that fulfil $N_L + 1 \leq C_L \leq p - N_R - 1$.

Sub-block II consists of α_{II} columns and $2\beta_{II}$ rows and builds the columns of the knot that fulfil $p - N_R \leq C_L \leq p - 1$.

Thus in our example where $N_L = 4$, $N_R = 5$ and $p = 31$ we obtain:

Sub-block I	builds all the columns	1	$\leq C_L \leq$	4	;
Main block	builds all the columns	5	$\leq C_L \leq$	25	;
Sub block II	builds all the columns	26	$\leq C_L \leq$	30	.

The bight indices of the knot are again calculated with the formula $pK^* - bL^* = I^*$ which we transform as before into:

$$(p - nb)K - bL = I, \qquad 0 \leq I \leq b, \qquad \text{(see page 38)}.$$

In general the values of α_I, α and α_{II} have to be *even* as otherwise we cannot get a repeat in horizontal direction; if any of the values is *odd* there can be *no* horizontal repeat.

The values of β_I, β and β_{II} have to be factors of the number of bights b, as otherwise we can not get a vertical repeat which is required for the knot to be cylindrical cyclic, which is normally the case.

For column-coded blocks the β values are 1 and for row-coded blocks the α values are 2.

In the formulae below we use the following additional notations:

$C_R =$ the column number of the knot as numbered from right to left;

$I_L =$ the left to right half-cycle starting at the left bight index I_L;

$I_R =$ the right to left half-cycle starting at the right bight index I_R;

$C_L^* =$ the column number in the repeating block as numbered from left to right;

$I_L^* =$ the string run from left to right in the repeating block.

The following equivalent coding relationships between the knot and its building blocks hold:

The coding of the intersection point indicated by C_L on I_L is equivalent to the coding of the intersection point indicated by C_L^* on I_L^* in the repeating block:

$1 \leq C_L \leq N_L \rightarrow$ sub-block I builds

$$C_{L_I}^* = |C_L|_{\alpha_I}$$

$$I_{L_I}^* = \left| I_L + \frac{C_L - C_{L_I}^*}{2} \right|_{\beta_I}$$

$N_L + 1 \leq C_L \leq p - 1 - N_R \rightarrow$ main block builds

$$C_L^* = |C_L - N_L|_\alpha$$

$$I_L^* = \left| I_L + \frac{C_L - C_L^* - N_L}{2} \right|_\beta$$

$p - N_R \leq C_L \leq p - 1 \rightarrow$ sub-block II builds

$$C_{L_{II}}^* = |C_L + N_R + 1 - p|_{\alpha_{II}}$$

$$I_{L_{II}}^* = \left| I_L + \frac{C_L - C_{L_{II}}^* + N_R + 1 - p}{2} \right|_{\beta_{II}}$$

The coding of the intersection point indicated by C_R on I_R is equivalent to the coding of the intersection point indicated by C_L^* on I_L^* in the repeating block:

$p - N_L \leq C_R \leq p - 1 \rightarrow$ sub-block I builds

$$C_{L_I}^* = |p - C_R|_{\alpha_I}$$

$$I_{L_I}^* = \left| I_R + \frac{p + C_R - C_{L_I}^*}{2} \right|_{\beta_I}$$

$N_R + 1 \leq C_R \leq p - 1 - N_L \rightarrow$ main block builds

$$C_L^* = |p - C_R - N_L|_\alpha$$

$$I_L^* = \left| I_R + \frac{p + C_R - C_L^* - N_L}{2} \right|_\beta$$

$1 \leq C_R \leq N_R \rightarrow$ sub-block II builds

$$C^*_{L_{II}} = |N_R + 1 - C_R|_{\alpha_{II}}$$
$$I^*_{L_{II}} = \left| I_R + \frac{N_R + 1 + C_R - C^*_{L_{II}}}{2} \right|_{\beta_{II}}$$

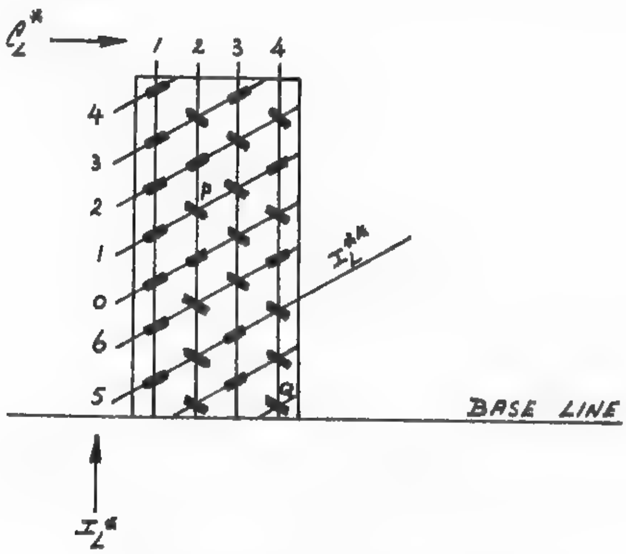


Diagram of the main block
for the example on page 60
(see Fig. 18)

Before giving a full knot example, we show how to determine whether a crossing is a *u* or *o* once we have computed a pair of (C,I)-values.

There are two cases (C_L, I_L) and (C_R, I_R) to consider. The first pair determines a crossing on an odd (L→R) half-cycle; the second pair determines a crossing on an even (R→L) half-cycle.

Examples using the main block diagram given above will make this clear:

A crossing in the knot indicated by intersection column C_L on half-cycle I_L (odd half-cycle) is equivalent, say, to crossing $P(C^*_L,\ I^*_L)$ in the main block ($C^*_L = 2$ and $I^*_L = 1$). Since the crossing is approached from L→R by this half-cycle we should deduce from the coding of $P(2, 1)$ that the crossing mark is to be read as **u**.

Similarly a crossing in the knot indicated by intersection column C_R on half-cycle I_R (even half-cycle) is equivalent, say, to crossing $Q(C^*_L,\ I^*_L)$ in the main block ($C^*_L = 4$ and $I^*_L = 3$). Since the crossing is approached from R→L by this half-cycle we should deduce from the coding of $Q(4, 3)$ that the crossing mark is to be read as **o**.

Example:

As will be obvious we do not have to draw the whole or even large parts of the knot diagram; it is sufficient to establish the knot pattern and the repeating blocks from that.

The diagrams on this and the facing page (Figs. 17 and 18) indicate the position of $I_L = I_0 = 0$ and the horizontal starting line (base line) for the knot pattern. Also in these diagrams is indicated the starting position and its value of I_L^* for each of the three repeating blocks; indicated as I_L^{**} in Fig. 17.

Sedgwick's Knot: $p/b = 31/21$ (see page 61 for diagrams).

Repeating main block : $\alpha = 4$; $\beta = 7$.
Repeating sub-block I : $\alpha_I = 4$; $\beta_I = 1$.
Repeating sub-block II : $\alpha_{II} = 4$; $\beta_{II} = 1$.
The bight index generating formula : $31K^* - 21L^* = I^* \rightarrow 10K - 21L = I$

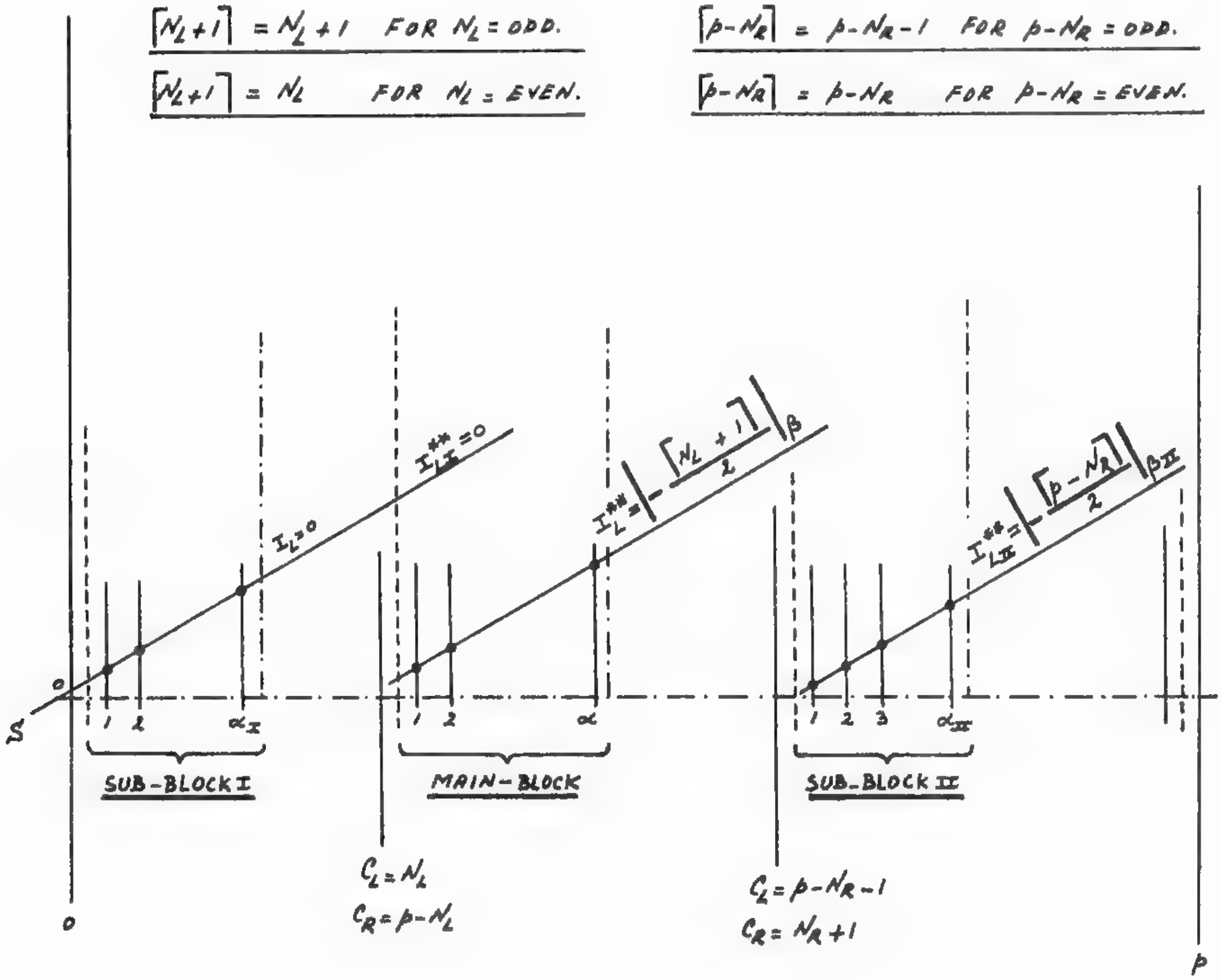
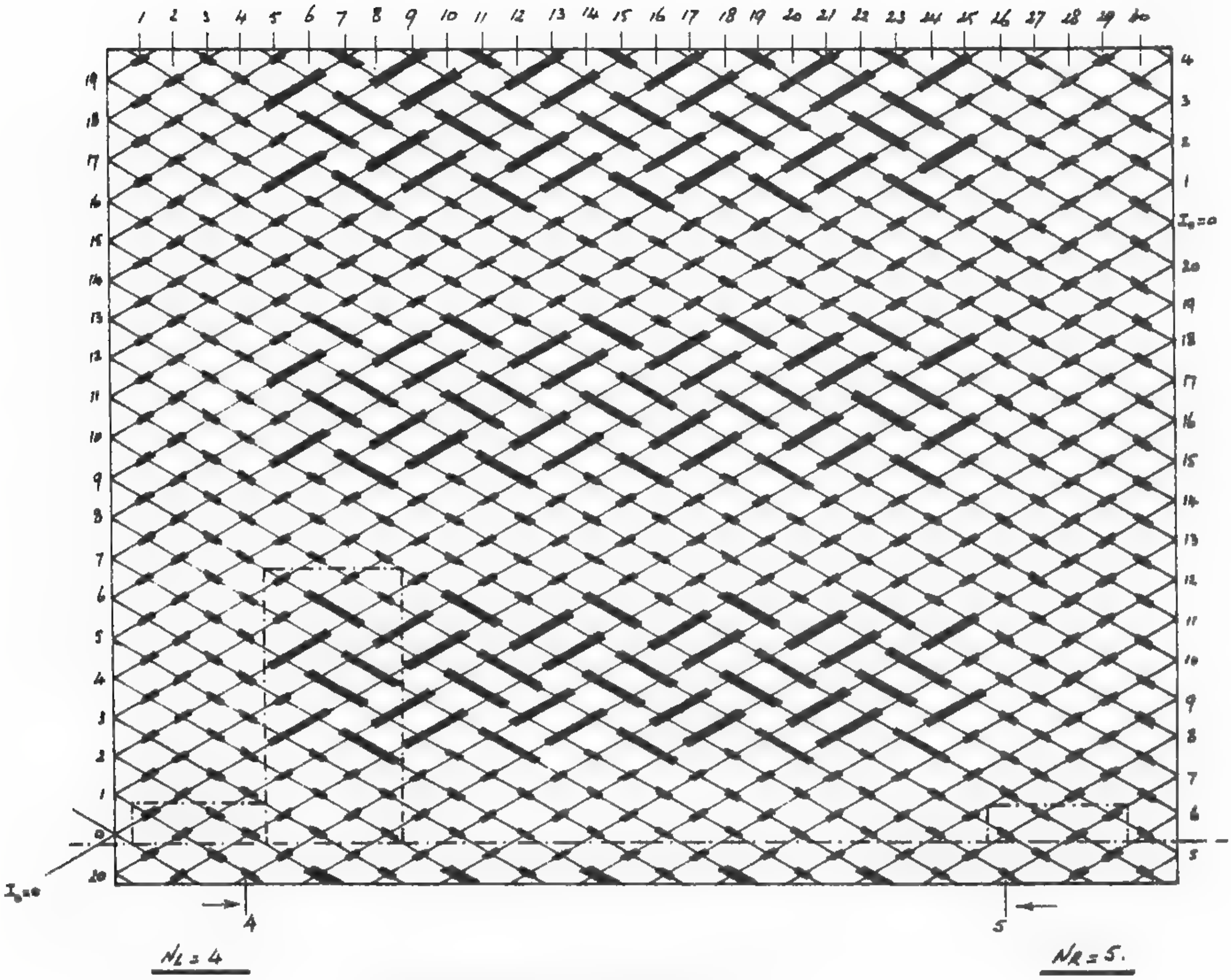
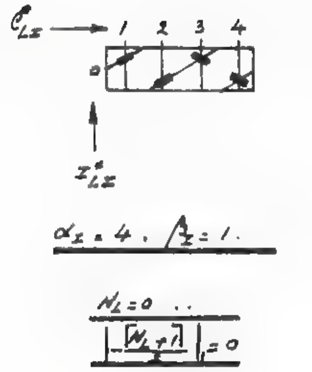


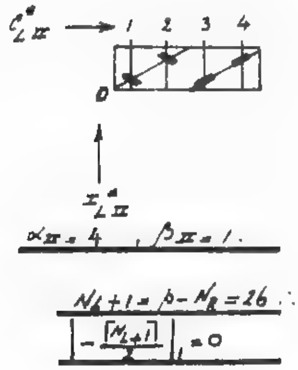
FIG. 17.



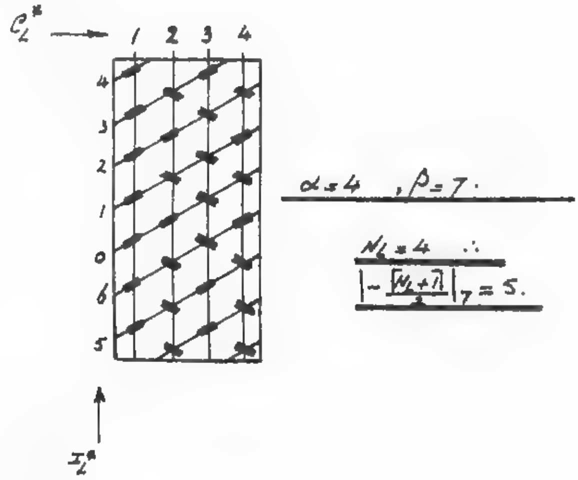
SEDGWICK'S KNOT.



SUB-BLOCK I.



SUB-BLOCK II.

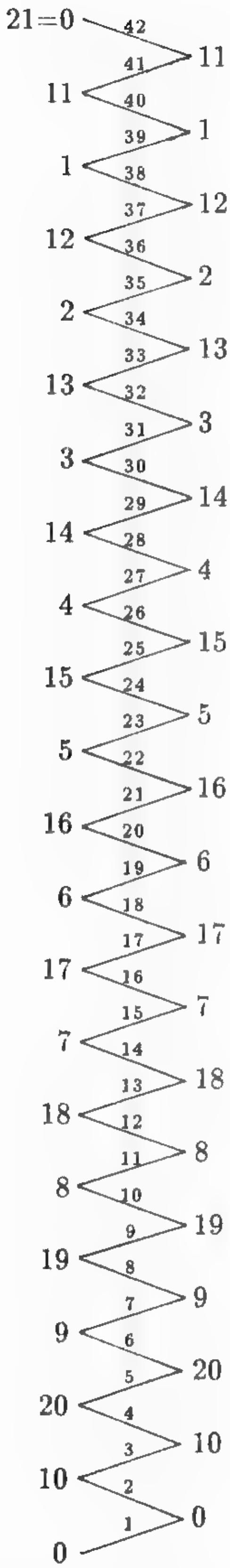


MAIN BLOCK.

FIG. 18.

1) $I_0 = 0$
2) $I_1 = 10$
3) $I_2 = 20$
4) $I_3 = 9$
5) $I_4 = 19$
6) $I_5 = 8$
7) $I_6 = 18$
8) $I_7 = 7$
9) $I_8 = 17$
10) $I_9 = 6$
11) $I_{10} = 16$
12) $I_{11} = 5$
13) $I_{12} = 15$
14) $I_{13} = 4$
15) $I_{14} = 14$
15) $I_{15} = 3$
17) $I_{16} = 13$
18) $I_{17} = 2$
19) $I_{18} = 12$
20) $I_{19} = 1$
21) $I_{20} = 11$

bight index numbers.



half-cycles and bight index numbers.

Intersection column C_L on I_L is coding-wise equivalent to:

Sub-block I: $1 \leq C_L \leq N_L$ so $1 \leq C_L \leq 4$.

$$C^*_{L_I} = |C_L|_{\alpha_I} = |C_L|_4$$

$$I^*_{L_I} = \left| I_L + \frac{C_L - C^*_{L_I}}{2} \right|_{\beta_I} = \left| I_L + \frac{C_L - C^*_{L_I}}{2} \right|_1 = 0.$$

Main block: $N_L + 1 \leq C_L \leq p - 1 - N_R,$ so $5 \leq C_L \leq 25$.

$$C^*_L = |C_L - N_L|_\alpha = |C_L - 4|_4$$

$$I^*_L = \left| I_L + \frac{C_L - C^*_L - N_L}{2} \right|_\beta = \left| I_L + \frac{C_L - C^*_L - 4}{2} \right|_7$$

Sub-block II: $p - N_R \leq C_L \leq p - 1,$ so $26 \leq C_L \leq 30$.

$$C^*_{L_{II}} = |C_L + N_R + 1 - p|_{\alpha_{II}} = |C_L - 25|_4$$

$$I^*_{L_{II}} = \left| I_L + \frac{C_L - C^*_{L_{II}} + N_R + 1 - p}{2} \right|_{\beta_{II}} = \left| I_L + \frac{C_L - C^*_{L_{II}} - 25}{2} \right|_1 = 0.$$

Intersection column C_R on I_R is coding-wise equivalent to:

Sub-block I: $p - N_L \leq C_R \leq p - 1,$ so $27 \leq C_R \leq 30$.

$$C^*_{L_I} = |p - C_R|_{\alpha_I} = |31 - C_R|_4$$

$$I^*_{L_I} = \left| I_R + \frac{p + C_R - C^*_{L_I}}{2} \right|_{\beta_I} = \left| I_R + \frac{31 - C_R - C^*_{L_I}}{2} \right|_1 = 0.$$

Main block: $N_R + 1 \leq C_R \leq p - 1 - N_L,$ so $6 \leq C_R \leq 26$.

$$C^*_L = |p - C_R - N_L|_\alpha = |27 - C_R|_4$$

$$I^*_L = \left| I_R + \frac{p + C_R - C^*_L - N_L}{2} \right|_\beta = \left| I_R + \frac{27 + C_R - C^*_L}{2} \right|_7$$

Sub-block II: $1 \leq C_R \leq N_R,$ so $1 \leq C_R \leq 5$.

$$C^*_{L_{II}} = |N_R + 1 - C_R|_{\alpha_{II}} = |6 - C_R|_4$$

$$I^*_{L_{II}} = \left| I_R + \frac{N_R + 1 + C_R - C^*_{L_{II}}}{2} \right|_{\beta_{II}} = \left| I_R + \frac{6 + C_R - C^*_{L_{II}}}{2} \right|_1 = 0.$$

1) 1st half-cycle L→R : **free run.**
2) 2nd half-cycle R→L : $I_R = 0$; $C_R = 21$.
   so $C^*_L = |27 - 21|_4 = 2$; $I^*_L = \left|0 + \frac{27+21-2}{2}\right|_7 = 2$ → **u.**
3) 3rd half-cycle L→R : $I_L = 10$; $C_L = 21$.

so $C_L^* = |21-4|_4 = 1;$ $I_L^* = \left|10 + \frac{21-1-4}{2}\right|_7 = 4$ $\to$ **o.**

4) 4th half-cycle R$\to$L : $I_R = 10$; $C_R = 11, 21$.

so $C_L^* = |27-11|_4 = 4;$ $I_L^* = \left|10 + \frac{27+11-4}{2}\right|_7 = 6$ $\to$ **u.**

$C_L^* = |27-21|_4 = 2;$ $I_L^* = \left|10 + \frac{27+21-2}{2}\right|_7 = 5$ $\to$ **o.**

So the coding is: **u–o.**

5) 5th half-cycle L$\to$R : $I_L = 20$; $C_L = 11, 21$.

so $C_L^* = |11-4|_4 = 3;$ $I_L^* = \left|20 + \frac{11-3-4}{2}\right|_7 = 1$ $\to$ **u.**

$C_L^* = |21-4|_4 = 1;$ $I_L^* = \left|20 + \frac{21-1-4}{2}\right|_7 = 0$ $\to$ **o.**

So the coding is: **u–o.**

6) 6th half-cycle R$\to$L : $I_R = 20$; $C_R = 1, 11, 21, 22$.

so $C_{L_{II}}^* = |6-1|_4 = 1;$ $I_{L_{II}}^* = 0$ $\to$ **o.**

$C_L^* = |27-11|_4 = 4;$ $I_L^* = \left|20 + \frac{27+11-4}{2}\right|_7 = 2$ $\to$ **o.**

$C_L^* = |27-21|_4 = 2;$ $I_L^* = \left|20 + \frac{27+21-2}{2}\right|_7 = 1$ $\to$ **o.**

$C_L^* = |27-22|_4 = 1;$ $I_L^* = \left|20 + \frac{27+22-1}{2}\right|_7 = 2$ $\to$ **u.**

So the coding is: **3o–u.**

7) 7th half-cycle L$\to$R : $I_L = 9$; $C_L = 1, 11, 21, 22$.

so $C_{L_I}^* = |1|_4 = 1;$ $I_{L_I}^* = 0$ $\to$ **o.**

$C_L^* = |11-4|_4 = 3;$ $I_L^* = \left|9 + \frac{11-3-4}{2}\right|_7 = 4$ $\to$ **o.**

$C_L^* = |21-4|_4 = 1;$ $I_L^* = \left|9 + \frac{21-1-4}{2}\right|_7 = 3$ $\to$ **o.**

$C_L^* = |22-4|_4 = 2;$ $I_L^* = \left|9 + \frac{22-2-4}{2}\right|_7 = 3$ $\to$ **u.**

So the coding is: **3o–u.**

8) 8th half-cycle R$\to$L : $I_R = 9$; $C_R = 1, 11, 12, 21, 22$.

so $C_{L_{II}}^* = 1;$ $I_{L_{II}}^* = 0$ $\to$ **o.**

$C_L^* = 4;$ $I_L^* = |2-(20-9)|_7 = 5$ $\to$ **o.**

$C_L^* = |27-12|_4 = 3;$ $I_L^* = \left|9 + \frac{27+12-3}{2}\right|_7 = 6$ $\to$ **o.**

$C_L^* = 2;$ $I_L^* = |1-11|_7 = 4$ $\to$ **o.**

$C_L^* = 1;$ $I_L^* = |2-11|_7 = 5$ $\to$ **u.**

So the coding is: **4o–u.**

9) 9th half-cycle L$\to$R : $I_L = 19$; $C_L = 1, 11, 12, 21, 22$.

so $C_{L_I}^* = 1;$ $I_{L_I}^* = 0$ $\to$ **o.**

$C_L^* = 3;$ $I_L^* = |(19-9)+4|_7 = 0$ $\to$ **u.**

$C_L^* = |12-4|_4 = 4;$ $I_L^* = \left|19 + \frac{12-4-4}{2}\right|_7 = 0$ $\to$ **u.**

$C_L^* = 1;$ $I_L^* = |10+3|_7 = 6$ $\to$ **o.**

$C_L^* = 2;$ $I_L^* = |10+3|_7 = 6$ $\to$ **u.**

So the coding is: **o–2u–o–u.**

10) 10th half-cycle R→L : $I_R = 19$; $C_R = 1, 2, 11, 12, 21, 22, 23$.

so $C^*_{L_{II}} = 1$; $I^*_{L_{II}} = 0$ → **o.**

$C^*_{L_{II}} = |6-2|_4 = 4$; $I^*_{L_{II}} = 0$ → **u.**

$C^*_L = 4$; $I^*_L = |10+5|_7 = 1$ → **u.**

$C^*_L = 3$; $I^*_L = |10+6|_7 = 2$ → **o.**

$C^*_L = 2$; $I^*_L = |10+4|_7 = 0$ → **u.**

$C^*_L = 1$; $I^*_L = |10+5|_7 = 1$ → **u.**

$C^*_L = |27-23|_4 = 4$; $I^*_L = |19 + \frac{27+23-4}{2}|_7 = 0$ → **o.**

So the coding is: **o–2u–o–2u–o.**

etc.

The full braiding algorithm for this knot is as follows:

1) L→R : **free run.**
2) R→L : **u.**
3) L→R : **o.**
4) R→L : **u–o.**
5) L→R : **u–o.**
6) R→L : **3o–u.**
7) L→R : **3o–u.**
8) R→L : **4o–u.**
9) L→R : **o–2u–o–u.**
10) R→L : **o–2u–o–2u–o.**
11) L→R : **3o–u–2o–u.**
12) R→L : **o–u–o–u–2o–u–o.**
13) L→R : **2o–u–3o–u–o.**
14) R→L : **o–2u–2o–u–o–2u–o.**
15) L→R : **2o–3u–2o–2u–o.**
16) R→L : **o–2u–o–u–o–3u–o–u.**
17) L→R : **2o–u–o–u–3o–u–o–u.**
18) R→L : **o–2u–o–u–2o–u–o–u–2o–u.**
19) R→L : **2o–3u–2o–u–2o–2u–o.**
20) R→L : **o–2u–2o–u–o–u–2o–2u–2o.**
21) L→R : **2o–2u–o–u–o–2u–o–u–o–u–o.**
22) R→L : **o–2u–4o–2u–2o–u–o–u–o–u.**
23) L→R : **2o–2u–o–2u–2o–u–o–2u–2o–u.**
24) R→L : **o–2u–2o–u–2o–u–o–3u–2o–2u.**
25) L→R : **2o–2u–2o–u–o–u–o–u–2o–2u–o–u.**
26) R→L : **o–2u–2o–u–o–u–o–2u–2o–u–o–u–o–u–o.**
27) L→R : **2o–2u–o–2u–2o–2u–2o–u–o–u–o–2u.**

28) R→L : **o–2u–2o–u–2o–2u–o–u–2o–2u–2o–u–o.**
29) L→R : **2o–2u–2o–2u–o–u–o–u–2o–2u–2o–2u.**
30) R→L : **o–2u–2o–2u–o–u–o–u–2o–3u–o–u–o–u–2o.**
31) L→R : **2o–2u–o–u–2o–u–2o–2u–2o–u–o–u–o–2u–o.**
32) R→L : **o–2u–2o–u–o–u–2o–2u–2o–u–o–u–2o–2u–2o.**
33) L→R : **2o–2u–o–3u–2o–u–o–u–o–u–2o–2u–o–2u–o.**
34) R→L : **o–2u–2o–u–3o–u–o–u–o–u–o–u–o–2u–2o–u–2o–u.**
35) L→R : **2o–2u–2o–2u–o–u–o–2u–2o–u–o–u–o–u–o–2u–2o.**
36) R→L : **o–2u–2o–u–o–u–2o–2u–2o–2u–2o–u–o–u–o–u–2o–u.**
37) L→R : **2o–2u–o–u–o–3u–2o–2u–o–u–2o–2u–2o–2u–2o.**
38) R→L : **o–2u–2o–2u–2o–u–2o–u–o–u–o–u–o–2u–2o–2u–2o–2u.**
39) L→R : **2o–2u–o–2u–3o–u–o–u–o–u–2o–u–2o–2u–o–2u–2o–u.**
40) R→L : **o–2u–2o–u–o–u–2o–u–o–2u–2o–2u–2o–u–o–u–o–u–2o–2u.**
41) L→R : **2o–2u–o–u–o–u–o–u–2o–2u–2o–u–o–u–o–u–o–u–o–2u–2o–u.**
42) R→L : **o–2u–2o–u–2o–2u–2o–2u–o–u–o–u–o–u–o–2u–2o–u–2o–2u.**

We will now look at the more general case where the coding of the knot is built up with many repeating blocks, each block building a *region* (see Fig. 19).

Region 1 is built up with block 1, region 2 with block 2, region 3 with block 3, etc. Thus the general region n is built up with block n.

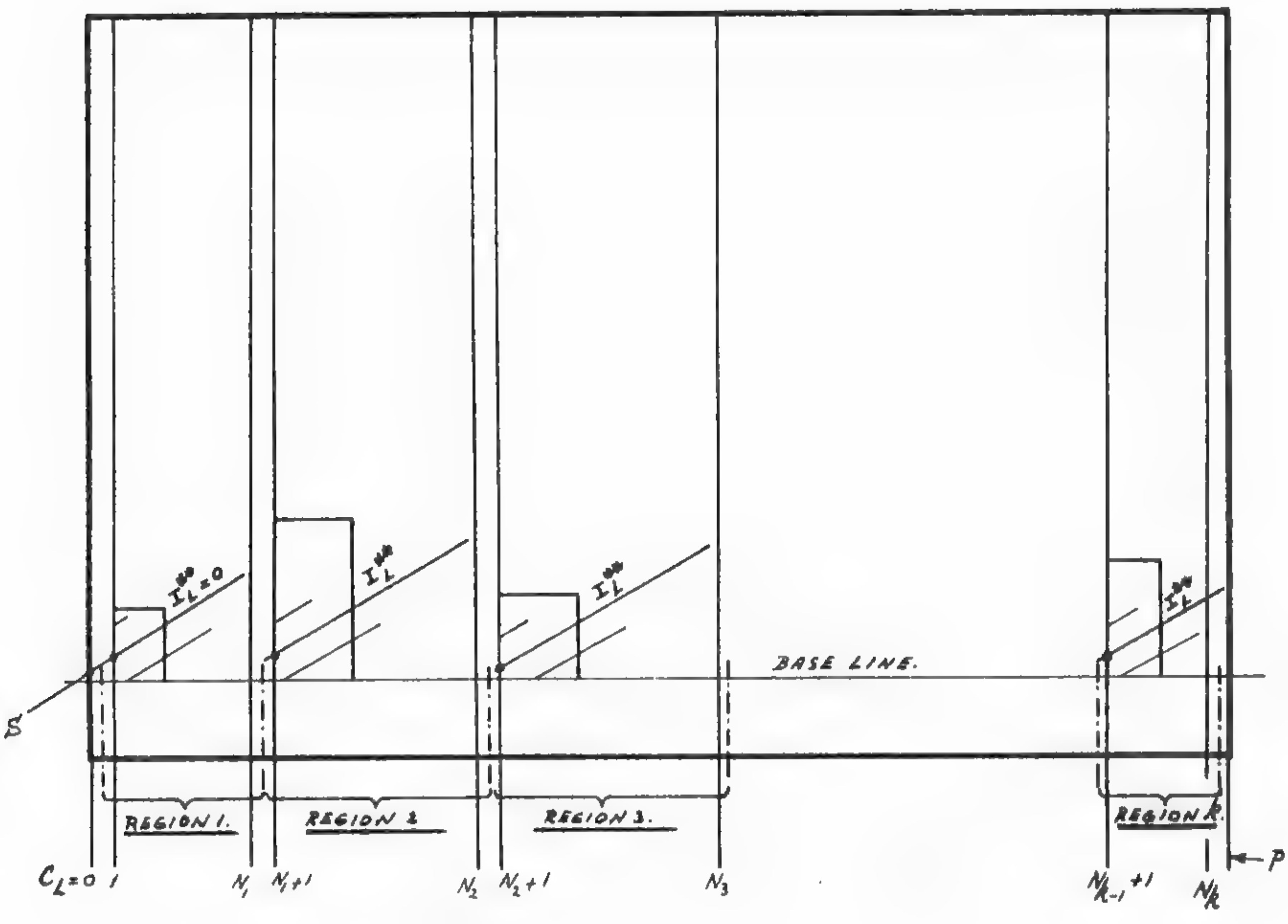
For each repeating block we have to calculate the value of I_L^{**}, being the first I_L^* with its left-hand starting point *above* the *base line* (see Fig. 19). The first left-hand starting point is the first crossing from the base line up on column $C_L^* = 1$. The base line is always positioned as indicated on the left in Fig. 20.

For region n with building block n we have:

$$N_{n-1} + 1 \le C_L \le N_n$$
$$p - N_n \le C_R \le p - N_{n-1} - 1$$
$$I_L^{**} = \left| - \frac{\lceil N_{n-1} + 1 \rceil}{2} \right|_{\beta_n} .$$

The coding of the intersection point indicated by C_L on I_L is equivalent to the coding of the point C_L^* on I_L^* of the building block n, whereby:

$$C_L^* = |C_L - N_{n-1}|_{\alpha_n}$$
$$I_L^* = \left| I_L + \frac{C_L - C_L^* - N_{n-1}}{2} \right|_{\beta_n}$$



$N_0 = 0.$

$N_k+1 = P.$

FOR REGION n : $N_{n-1}+1 \leq C_L \leq N_n$; $I_L^{**} = \left| -\frac{[N_{n-1}+1]}{2} \right|_{\beta_n}$

$[N_{n-1}+1] = N_{n-1}+1$ FOR N_{n-1} ODD.

$[N_{n-1}+1] = N_{n-1}$ FOR N_{n-1} EVEN.

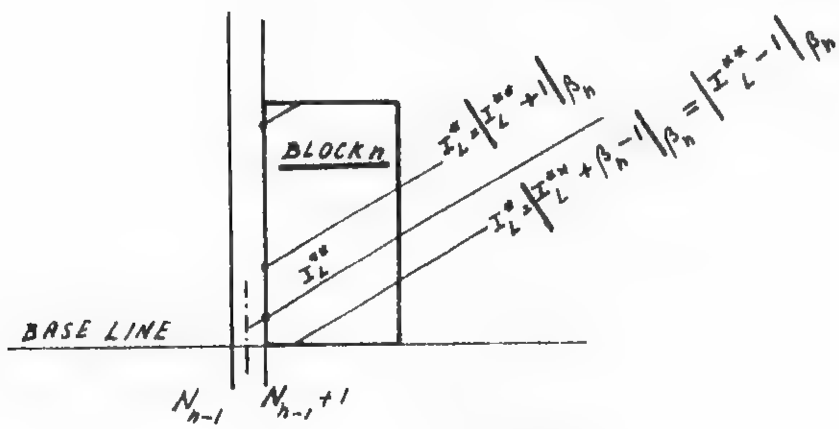


FIG. 10

***The coding of the intersection point indicated by C_R on I_R is equivalent to the coding of the point C_L^* on I_L^* of the building block* n*, whereby*:

$$C_L^* = |p - C_R - N_{n-1}|_{\alpha_n}$$

$$I_L^* = \left| I_R + \frac{p + C_R - C_L^* - N_{n-1}}{2} \right|_{\beta_n}$$

It is in general however easier to describe the coding of the building block according to one of the following two methods:

(A) describe for *each column* the coding ***row-wise***.

(B) describe for *each column* the coding ***entry-wise*** from the base line upward. Refer to the diagrams in Figures 19 and 20.

In Fig. 20 the row-wise description of the coding for each column is indicated in the top centre diagram while the entry-wise description of the coding for each column is indicated in the bottom centre diagram.

It should be pointed out here that these coding descriptions do not depend on whether N_{n-1} is odd or even; the two odd and even building blocks presented here are to show the two possible building block forms, not the coding entry methods.

Coding description method (A)

For region n the following relationships hold:

$$N_{n-1} + 1 \le C_L \le N_n$$

$$p - N_n \le C_R \le p - N_{n-1} - 1$$

$\lceil N_{n-1} + 1 \rceil = N_{n-1} + 1$ for N_{n-1} is *odd*.
$\lceil N_{n-1} + 1 \rceil = N_{n-1}$ for N_{n-1} is *even*.

$$I_L^{**} = \left| -\frac{\lceil N_{n-1} + 1 \rceil}{2} \right|_{\beta_n}$$

$$I_L^* = \left| -\frac{N_{n-1} + C_L^* - \Re}{2} \right|_{\beta_n}$$

where $\Re$ is the row number.

***The coding of the intersection point indicated by C_L on I_L is equivalent to the coding of the point indicated by $\Re$ on C_L^* of the building block* n*, whereby*:

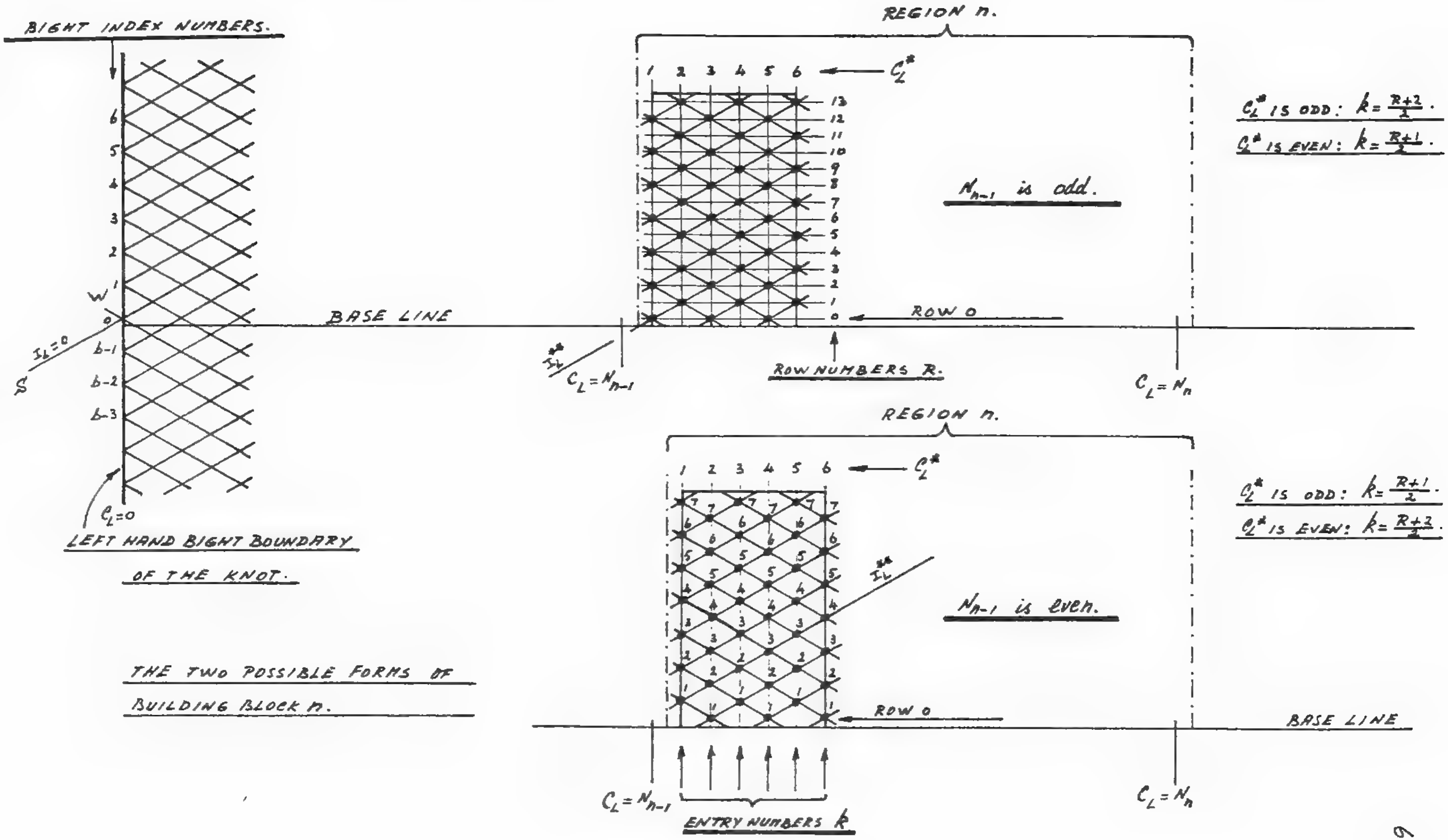
$$C_L^* = |C_L - N_{n-1}|_{\alpha_n}$$

$$\Re = |2I_L + C_L|_{2\beta_n}$$

***The coding of the intersection point indicated by C_R on I_R is equivalent to the coding of the point indicated by $\Re$ on C_L^* of the building block* n*, whereby*:

$$C_L^* = |p - C_R - N_{n-1}|_{\alpha_n}$$

$$\Re = |2I_R + C_R + p|_{2\beta_n}$$



THE TWO POSSIBLE FORMS OF BUILDING BLOCK n.

FIG. 20.

Coding description method (B)

For region n the following relationships hold:

$$N_{n-1} + 1 \leq C_L \leq N_n$$
$$p - N_n \leq C_R \leq p - N_{n-1} - 1$$

$\lceil N_{n-1} + 1 \rceil = N_{n-1} + 1$ for N_{n-1} is *odd*.
$\lceil N_{n-1} + 1 \rceil = N_{n-1}$ for N_{n-1} is *even*.

$$I_L^{**} = \left| -\frac{\lceil N_{n-1} + 1 \rceil}{2} \right|_{\beta_n}$$
$$I_L^{*} = \left| -\frac{\lceil N_{n-1} + C_L^{*} \rceil}{2} + k - 1 \right|_{\beta_n}$$

where $\lceil N_{n-1} + C_L^* \rceil = N_{n-1} + C_L^*$ for $(N_{n-1} + C_L^*)$ is *even*
$\lceil N_{n-1} + C_L^* \rceil = N_{n-1} + C_L^* - 1$ for $(N_{n-1} + C_L^*)$ is *odd*

The coding of the intersection point indicated by C_L on I_L is equivalent to the coding of the point indicated by entry k on C_L^* of the building block n, whereby:

$$C_L^* = |C_L - N_{n-1}|_{\alpha_n}$$
$$k = \left| I_L + \frac{\lceil C_L \rceil}{2} + 1 \right|_{\beta_n}$$

where $\lceil C_L \rceil = C_L$ for C_L is *even*
$\lceil C_L \rceil = C_L - 1$ for C_L is *odd*

The coding of the intersection point indicated by C_R on I_R is equivalent to the coding of the point indicated by entry k on C_L^* of the building block n, whereby:

$$C_L^* = |p - C_R - N_{n-1}|_{\alpha_n}$$
$$k = \left| I_R + \frac{\lceil p + C_R \rceil}{2} + 1 \right|_{\beta_n}$$

where $\lceil p + C_R \rceil = p + C_R$ for $(p + C_R)$ is *even*
$\lceil p + C_R \rceil = p + C_R - 1$ for $(p + C_R)$ is *odd*

Method (A) is generally preferred for *row-coded* building blocks whereas method (B) is preferred for *non column-non row coded* building blocks.

For *column coded* building blocks we only needed to calculate the C_L^* values since for these building blocks $\beta_n = 1$ and thus $I_L^* = 0$.

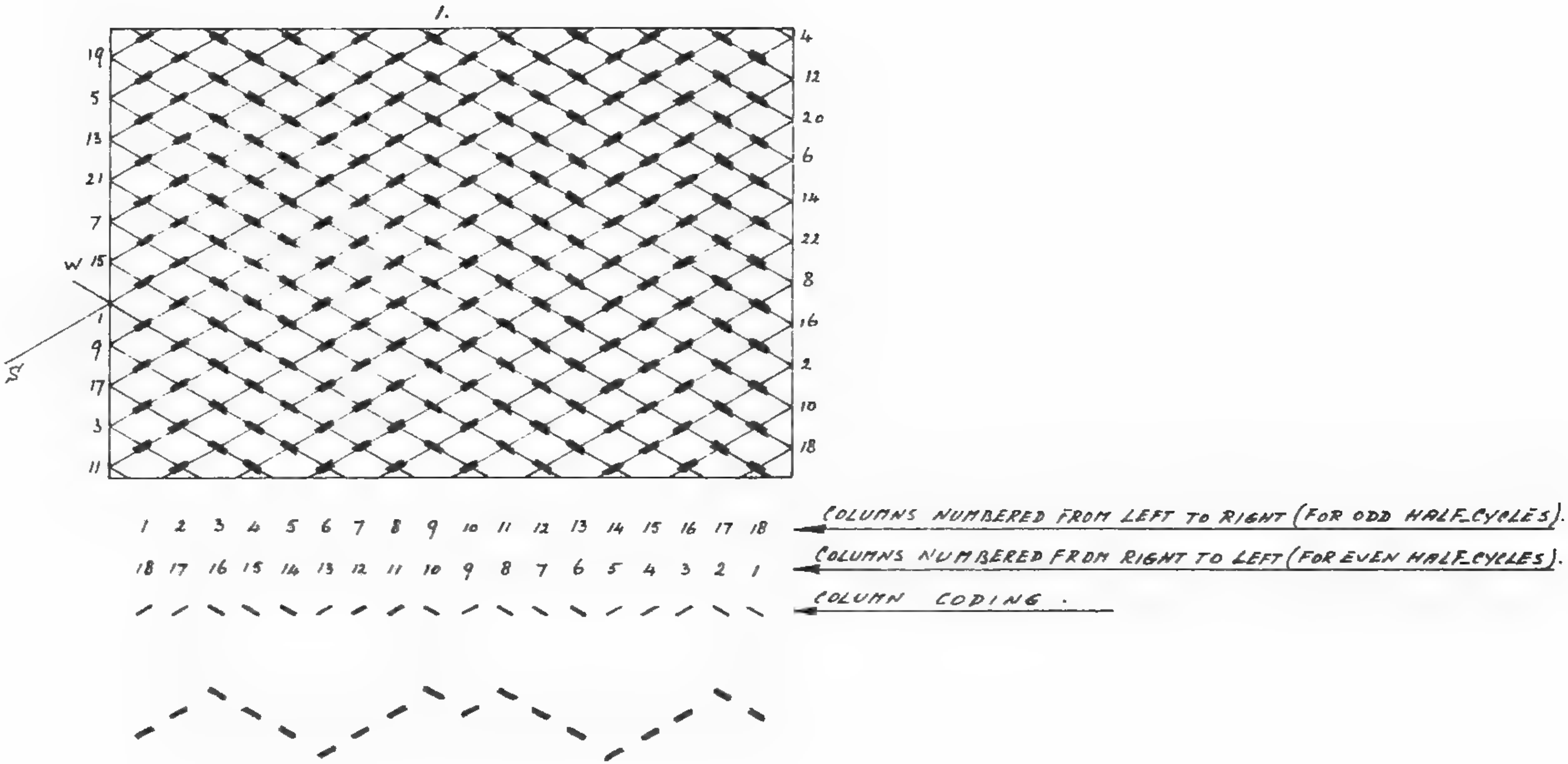
For *row-coded* building blocks $\alpha_n = 2$ as already mentioned on page 57.

The value of α_n, must be *even* to make a horizontal repeat possible; the only exception is when $\alpha_n = N_n - N_{n-1}$. Only in this case can α_n be odd since then we have no horizontal repeats in region n.

Exercises

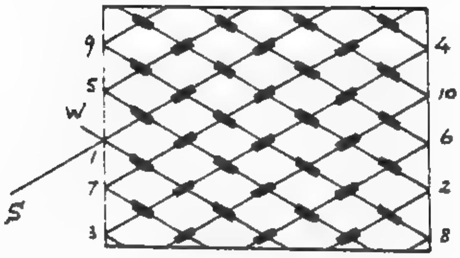
In the following pages we give the grid diagrams of fortytwo knots. The reader is invited to use one or other of the methods given in this book in order to obtain their algorithm tables. Solutions may be found after the Appendix.

List of Exercises:



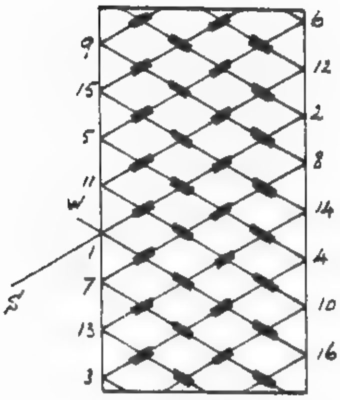
FAN KNOT $P/b = 19/11$.

2.



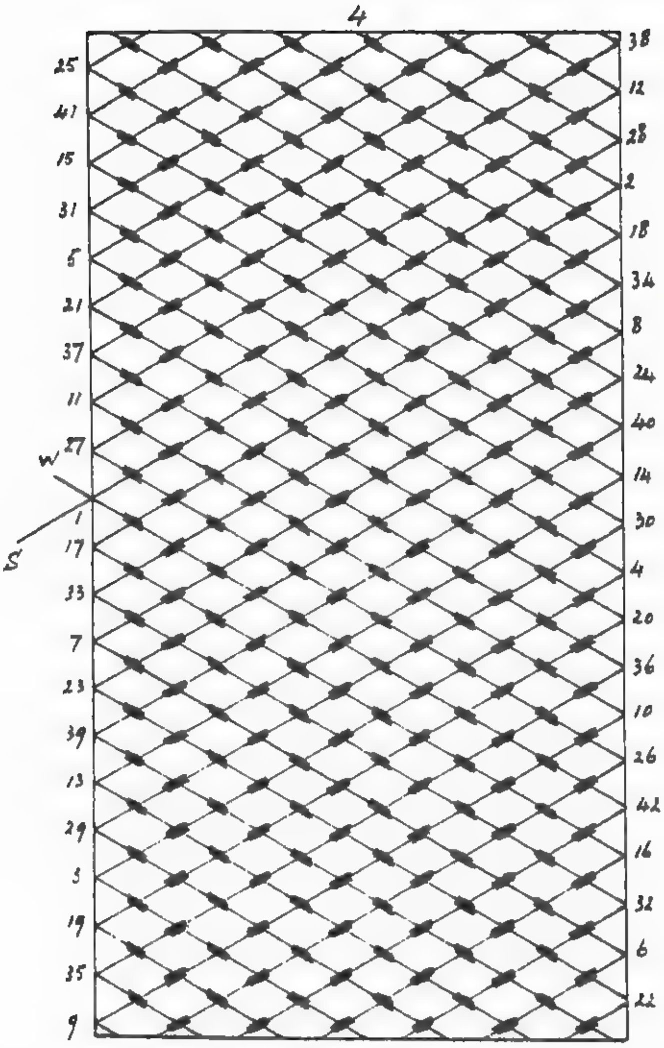
8/5 TURK'S HEAD.

3.



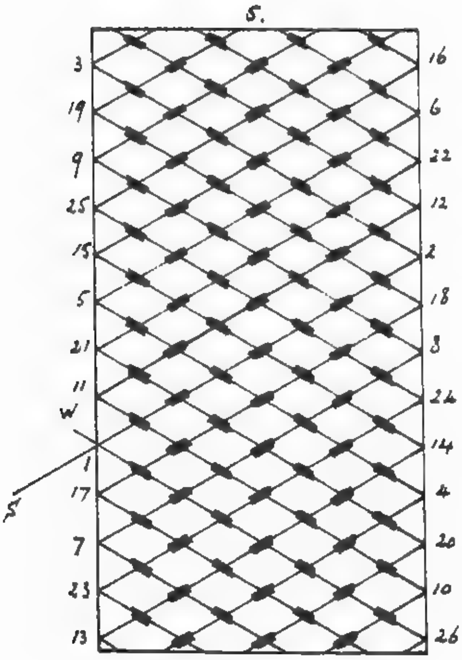
5/8 TURK'S HEAD.

4



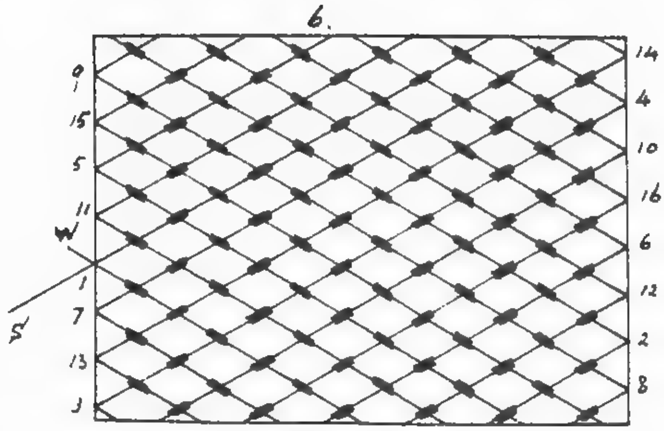
13/21 TURK'S HEAD.

5.



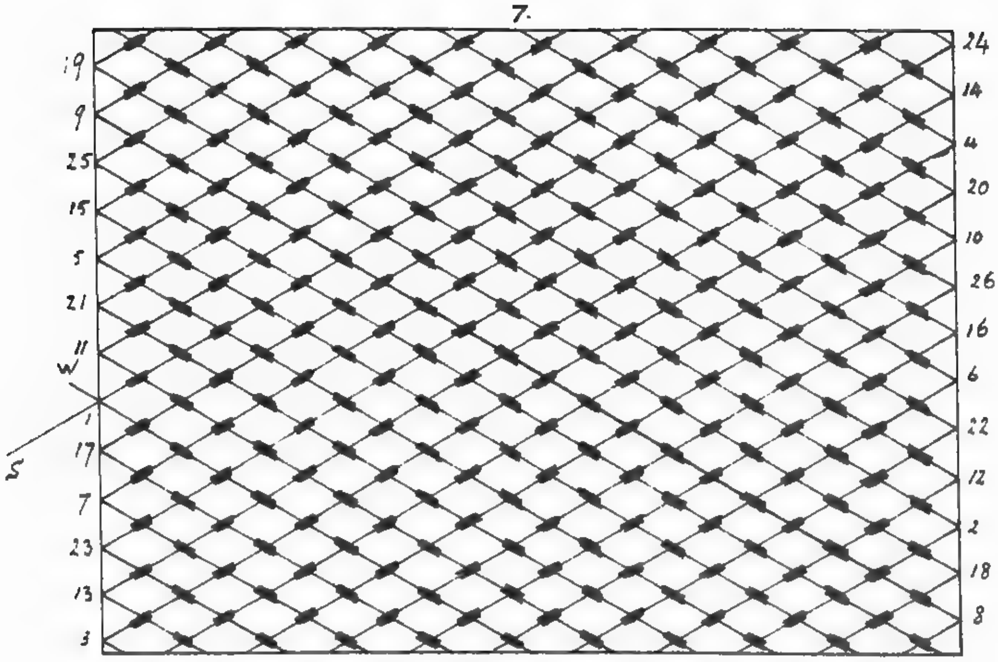
8/13 TURK'S HEAD.

6.



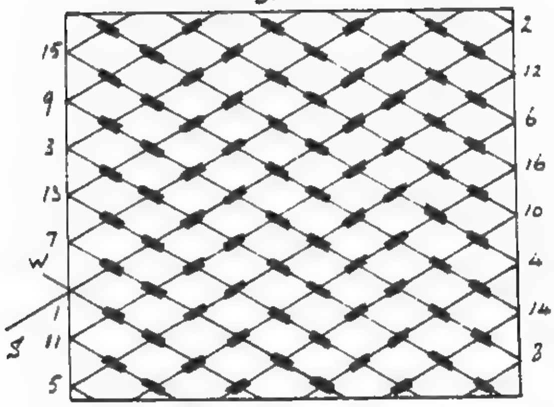
13/8 TURK'S HEAD.

7.



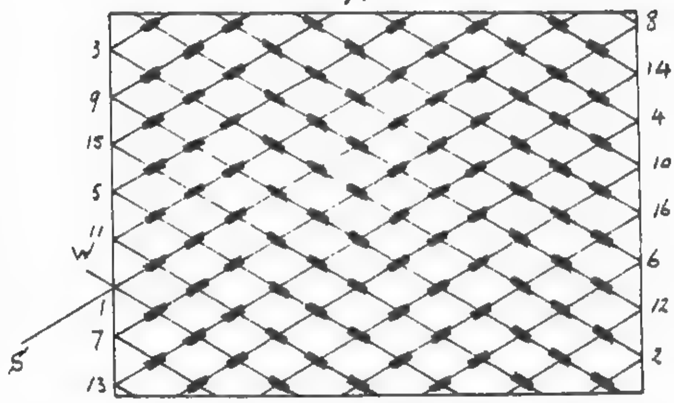
21/13 TURK'S HEAD.

8.



11/8 HEAD HUNTER'S KNOT.

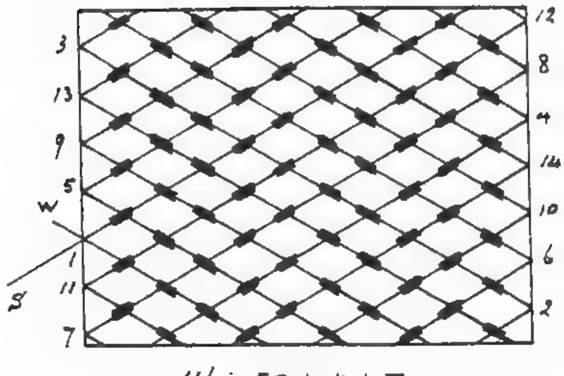
9.



13/8 GAUCHO

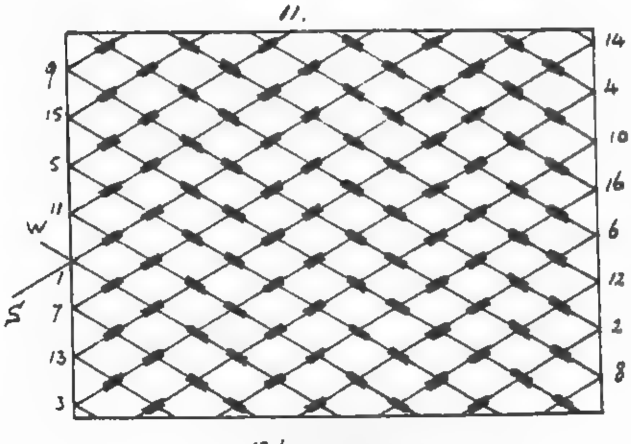
3 PASS.

10.



11/7 FAN KNOT.

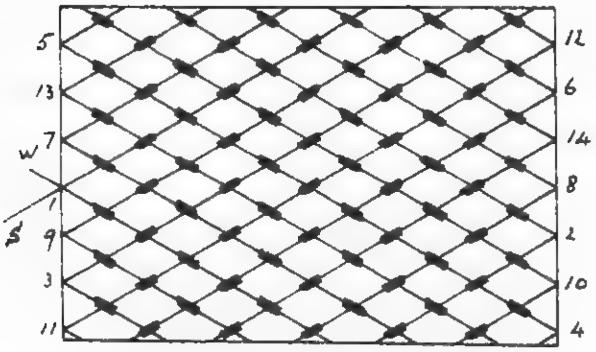
11.



13/8 GAUCHO

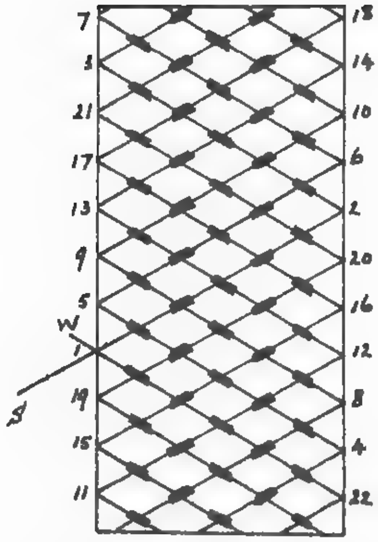
2 PASS.

12.



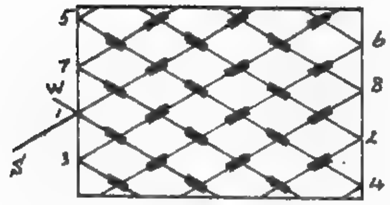
TURK'S HEAD p/b = 12/7.

13.



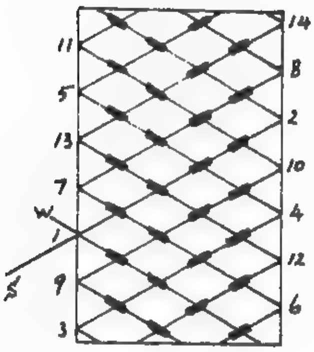
TURK'S HEAD p/b = 6/11

14.



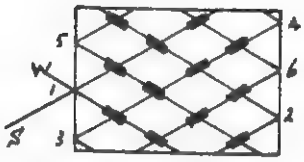
TURK'S HEAD p/b = 7/4

15.



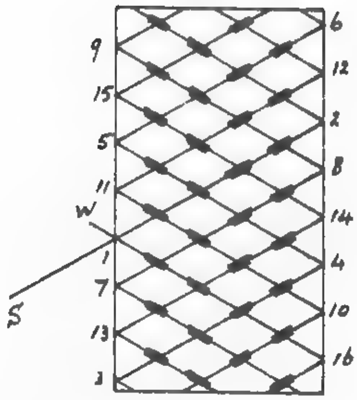
TWO-PASS RING KNOT
p/b = 5/7

16.



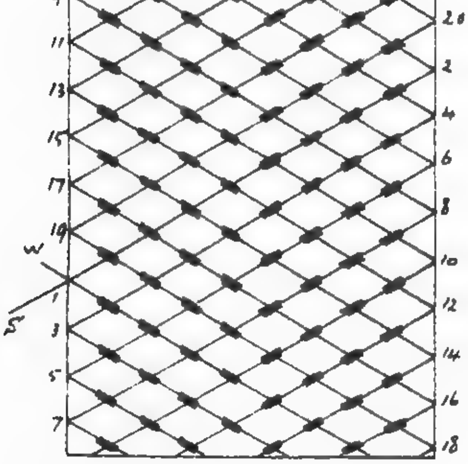
TWO-PASS RING KNOT
p/b = 5/3

17.



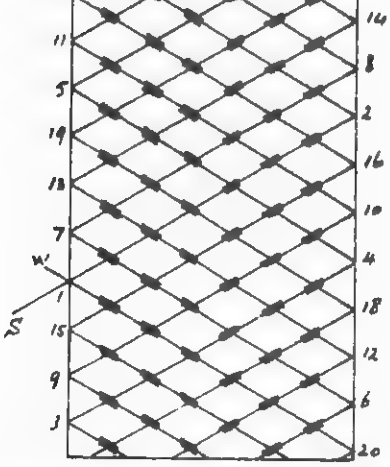
TWO-PASS RING KNOT.
p/b = 5/8

18.



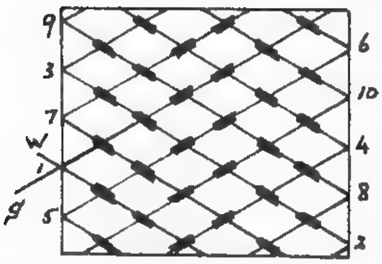
FOUR-PASS RING KNOT p/b = 9/10

19.



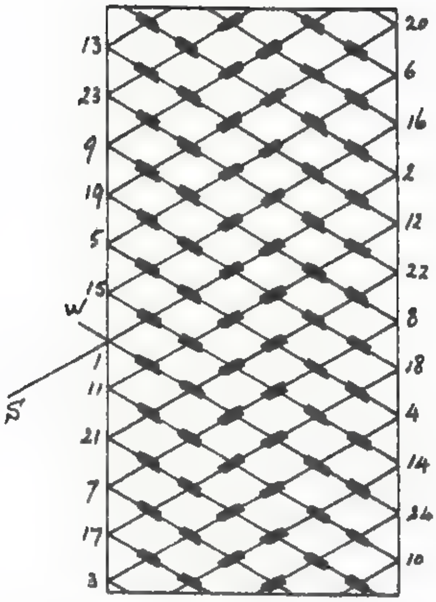
THREE-PASS RING KNOT p/b = 7/10

20.



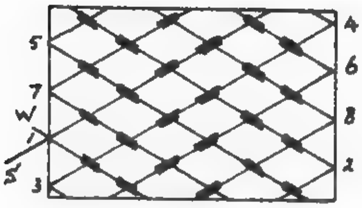
HEADHUNTER'S KNOT P/b = 7/5

21.



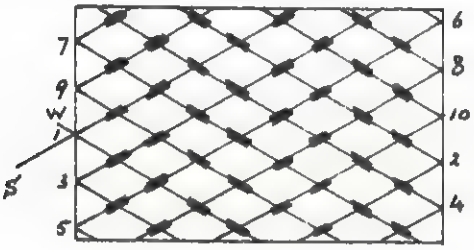
HEAD HUNTER'S KNOT P/b = 7/12

22.



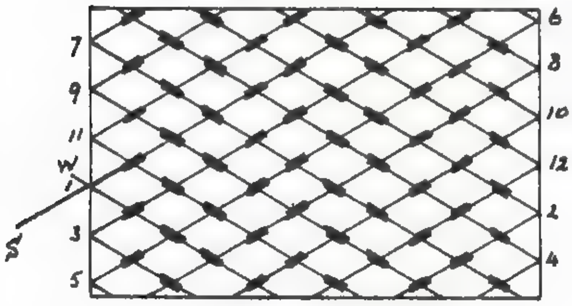
HEAD HUNTER'S KNOT P/b = 7/4

23



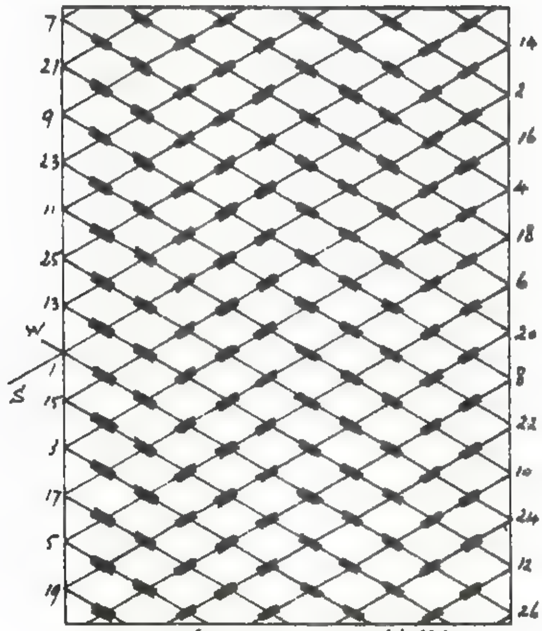
GAUCHO KNOT P/b = 9/5

24.



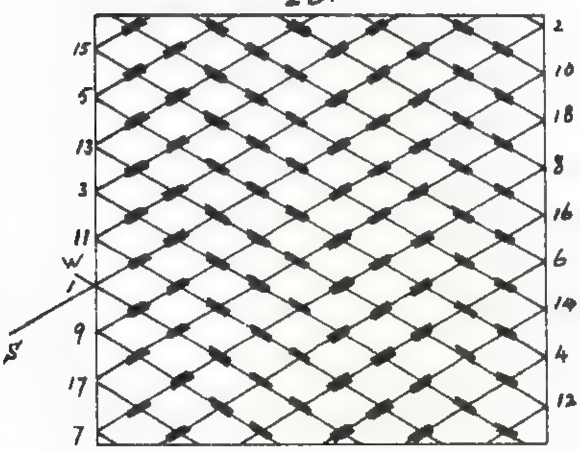
BOTÓN ORIENTAL P/b = 11/6

25.



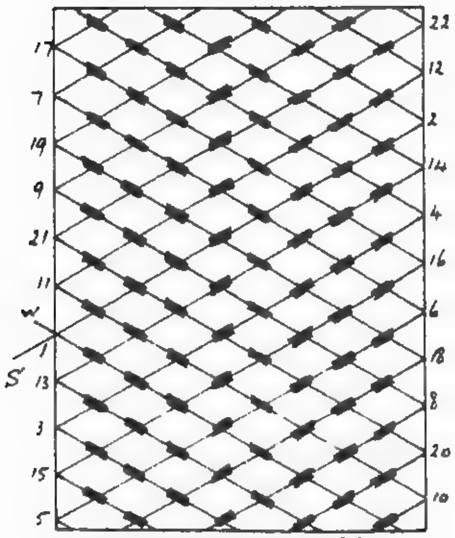
BOTÓN ORIENTAL P/b = 11/13

26.



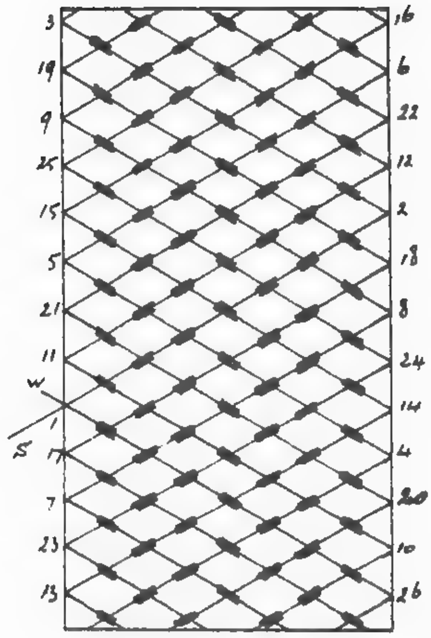
BOTÓN ORIENTAL P/b = 11/9

27.



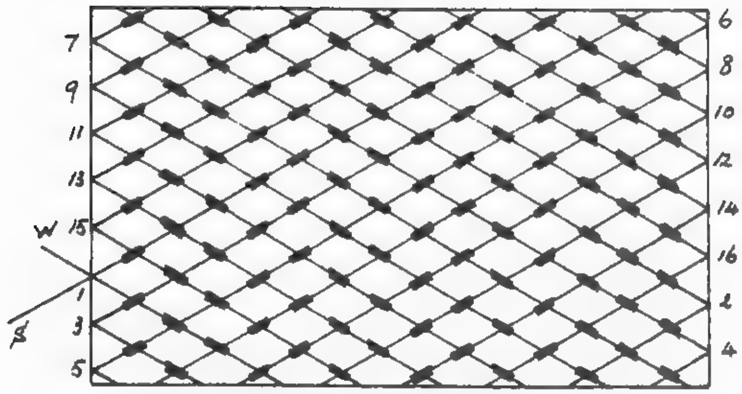
GINFER KNOT P/b = 9/11

28.



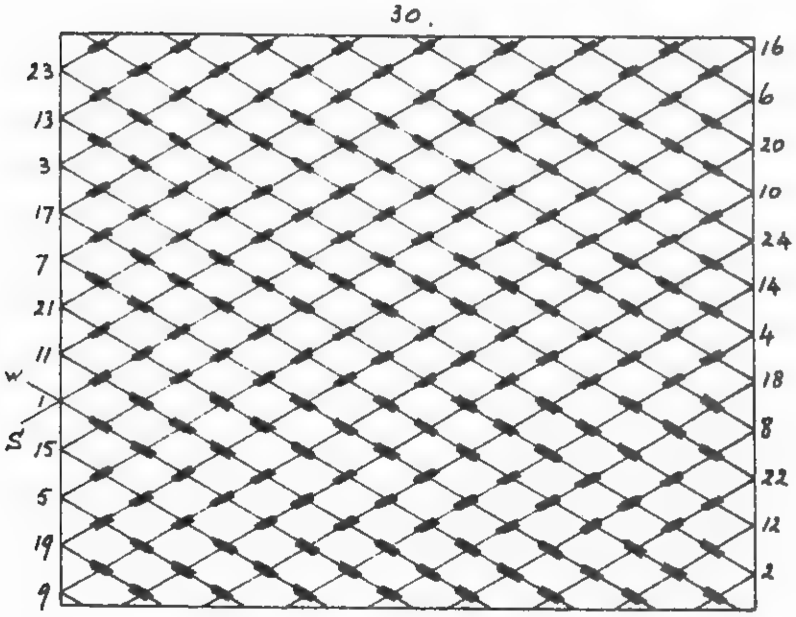
SAMUEL KNOT P/b = 8/13.

29.



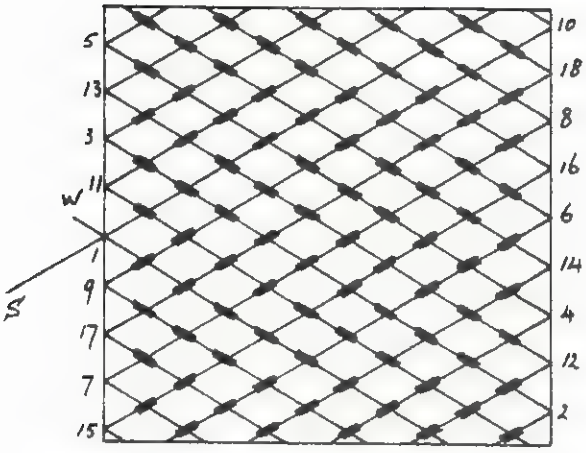
AZTEC-FAN KNOT P/b = 15/8.

30.



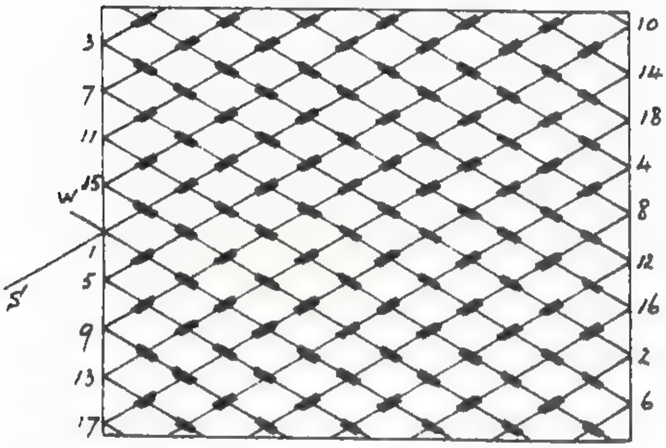
PERFECT THREE-PASS HERRING BONE KNOT P/b = 17/12.

31.



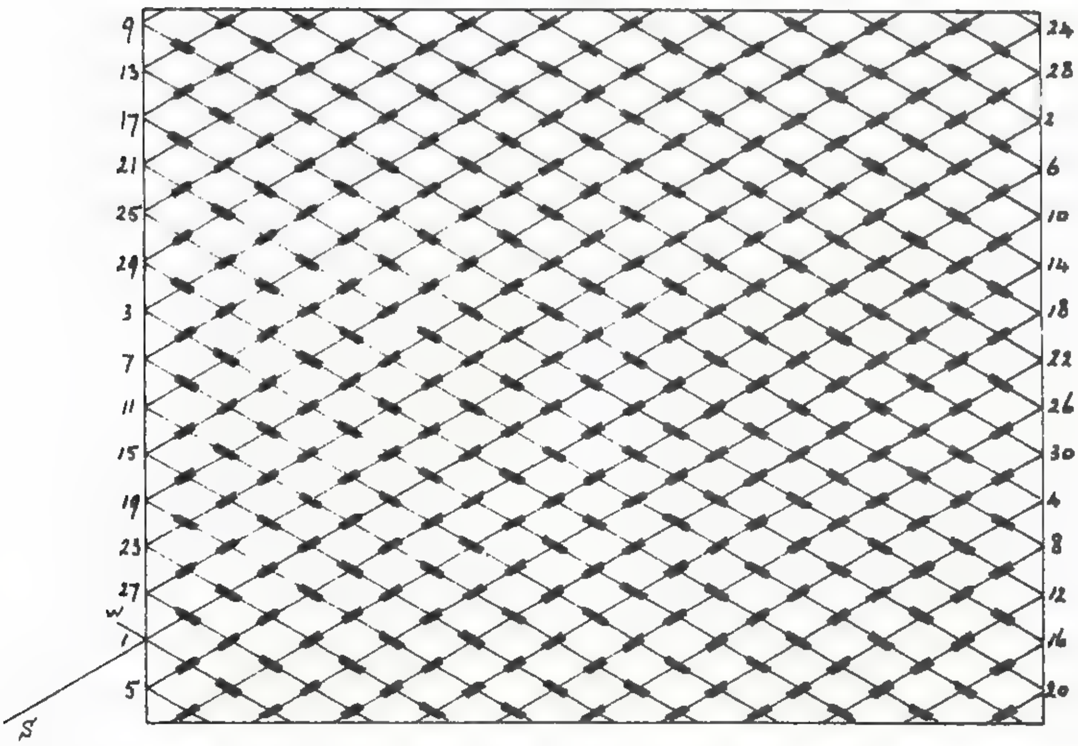
PERFECT THREE-PASS HERRING BONE KNOT $P/b = 11/9$

32.



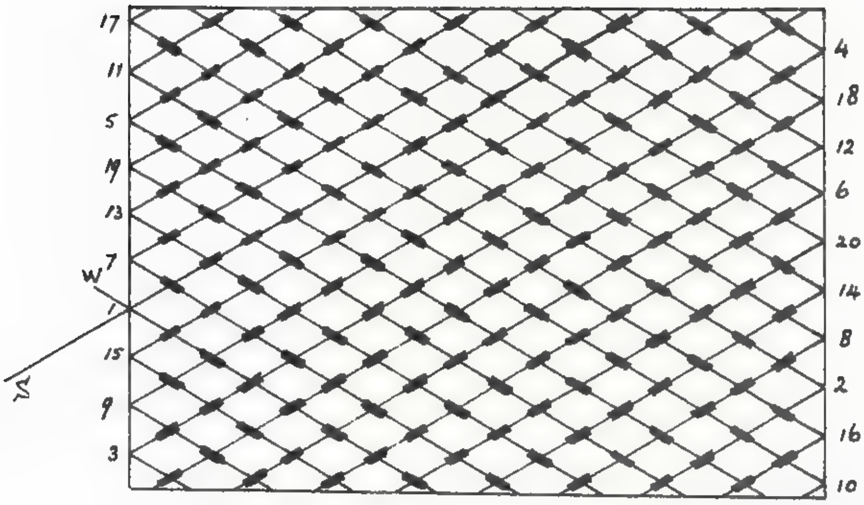
BEVERLEY KNOT $P/b = 13/9$.

33.



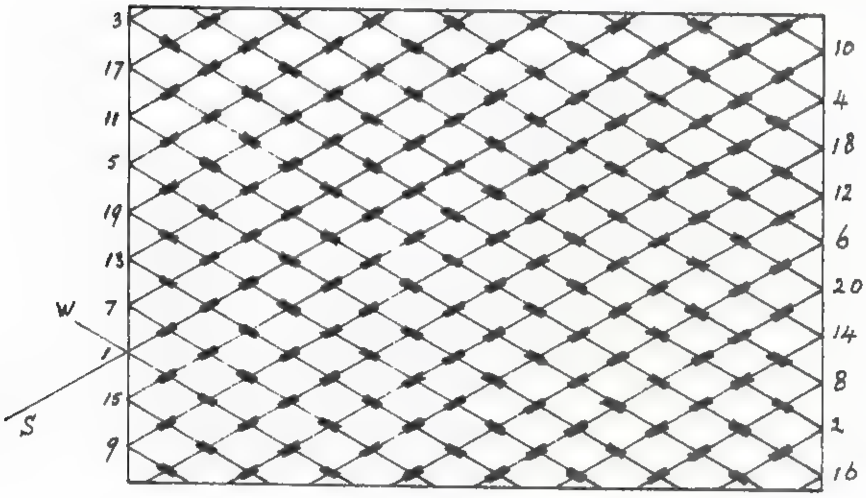
SLOW HELIX KNOT TYPE I $P/b = 23/15$ (BRAID OVER COLLAR SINCE ENDS PARTLY CLOSE)

34.



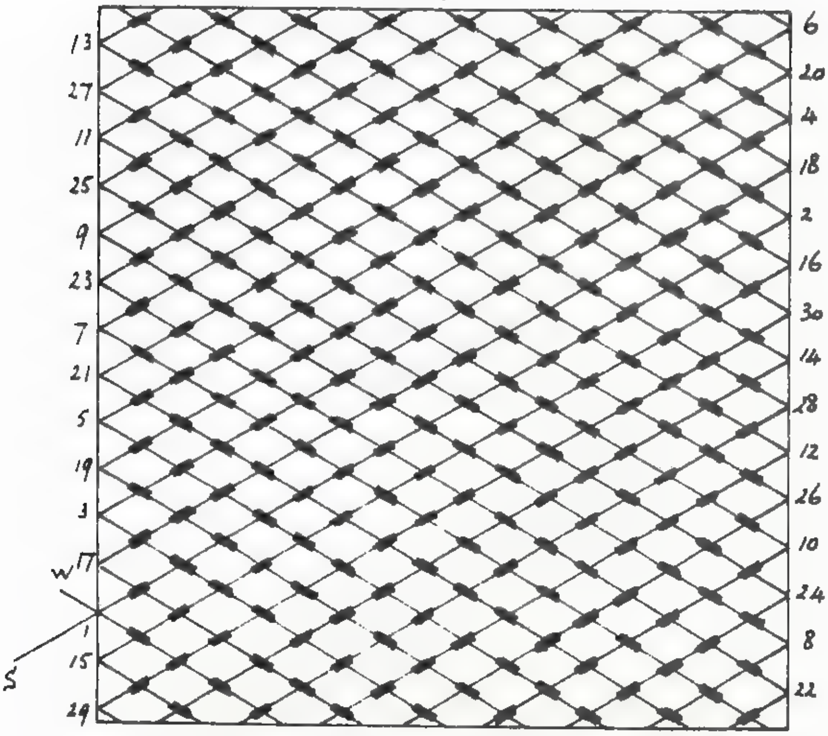
SLOW HELIX KNOT TYPE I $P/b = 17/10$ (BRAID OVER COLLAR).

35.



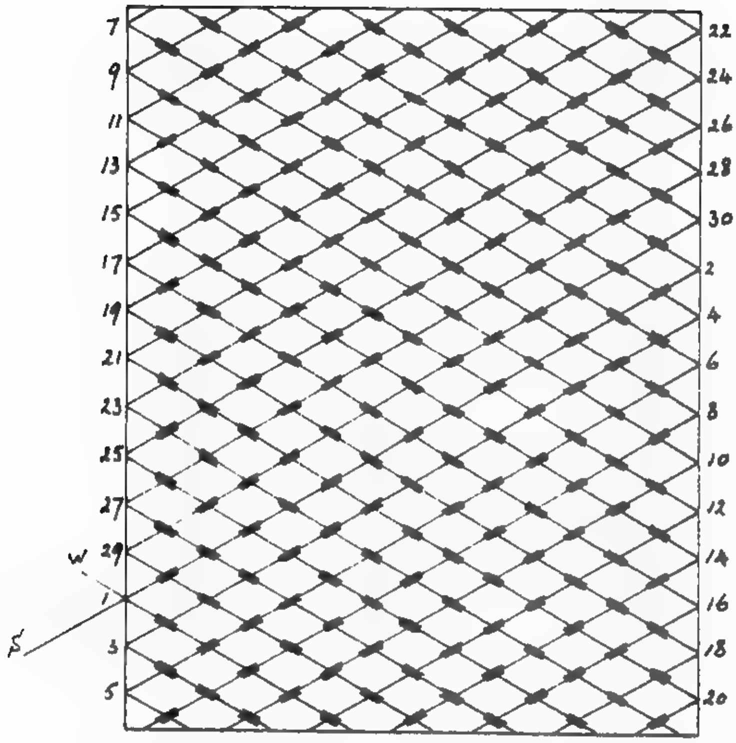
FAST HELIX KNOT $P/b = 17/10$ (BRAID OVER COLLAR).

36.



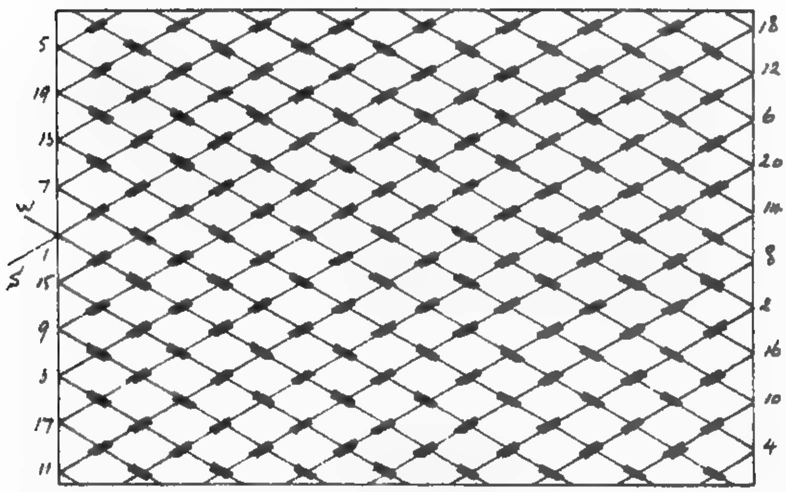
BASKET WEAVE KNOT TYPE 6n+5 $P/b = 17/15$

37.



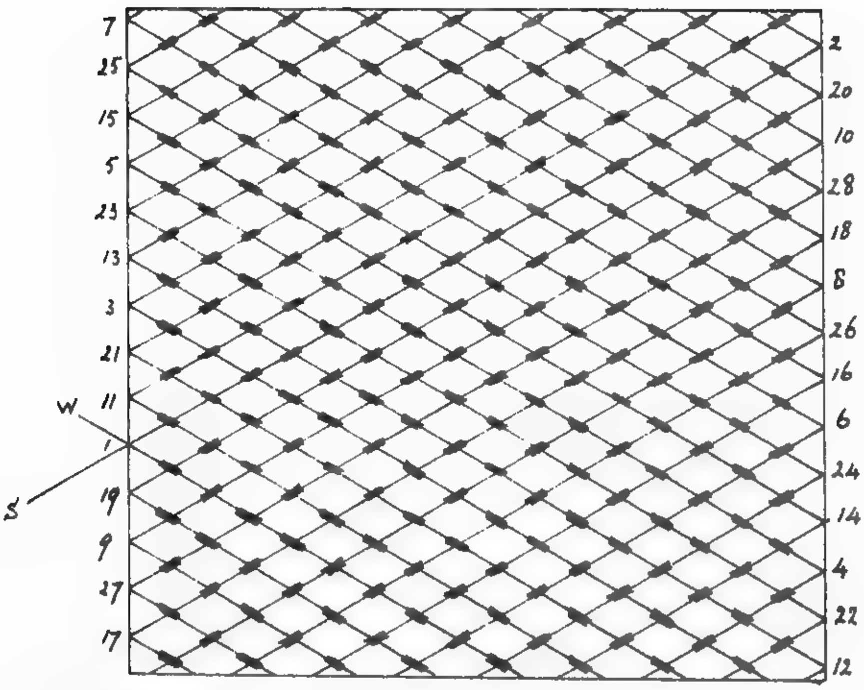
BASKET WEAVE KNOT TYPE 6h+2 P/b = 14/15.

38.



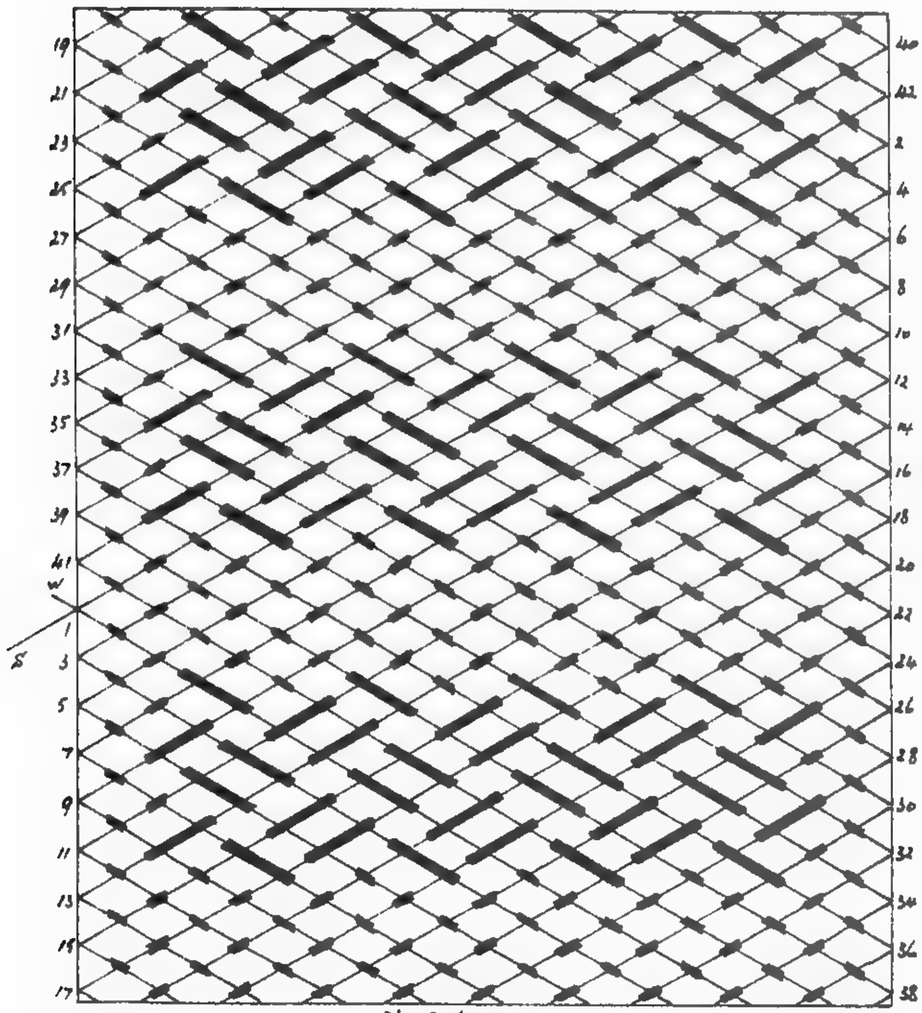
ROW CODED. 01 KNOT P/b = 17/10

39.



ROW CODED. P/b = 17/14. O2 KNOT

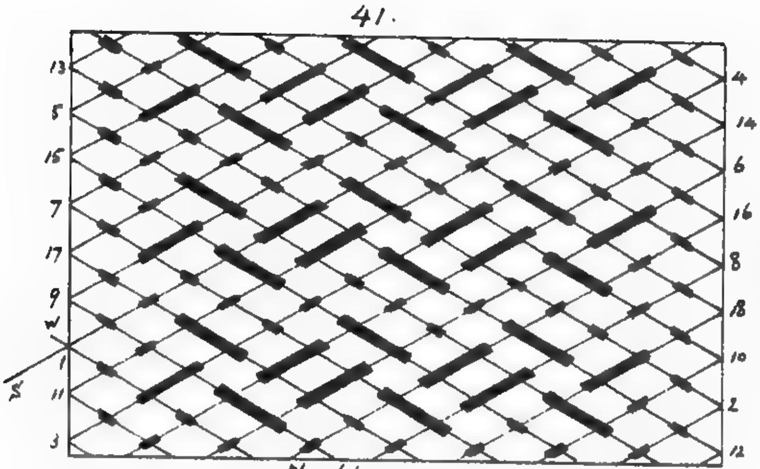
40.



p = multiple of 4.
b = multiple of 7.

P/b = 20/21 O5 KNOT

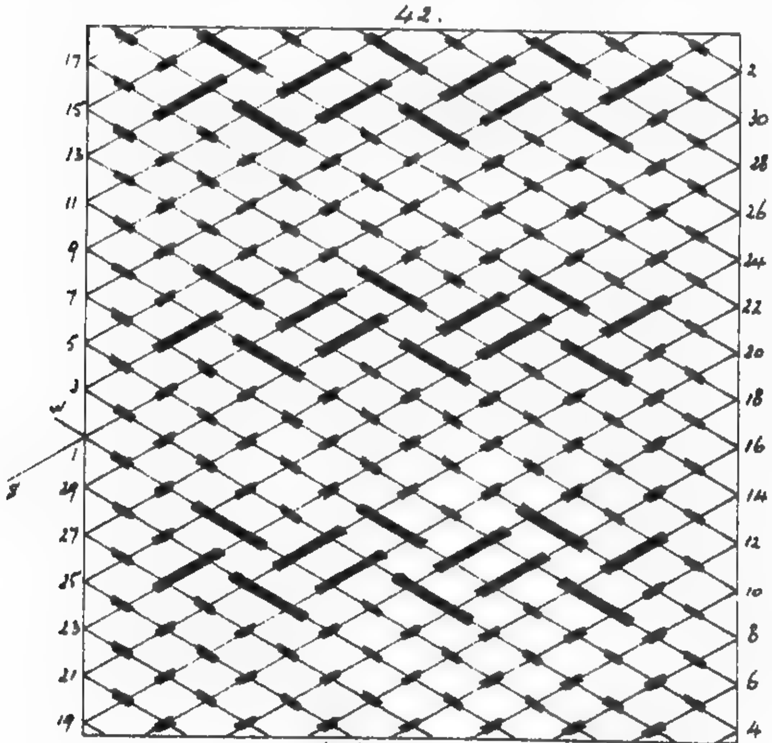
41.



p/b = 16/9 O3 KNOT

p =
multiple of 4.
b =
multiple of 3.

42.



p/b = 16/15 O4 KNOT

p =
multiple of 4.
b =
multiple of 5.

Appendix

RELATIONSHIP BETWEEN NUMBER OF BIGHTS AND KNOT DIAMETER

When designing a knot which has to fit around a given object, it is necessary to estimate the required number of bights which will ensure a satisfactory fit. The following formulae may be used to achieve this.

Although the thickness and rigidity of the string used, and the coding, do have an effect on the final diameter of a braided knot, the principal influencing variables are the number of bights ($= b$) and the width of the string ($= w$). If d is the diameter of the object or braid over which the braided knot has to fit, the following empirical formulae will give a good approximation.

If the crossings adjacent to one of the bight edges of the knot are 1 under/1 over or vice-versa, a normal braid tightness may be obtained using b calculated from:

$$b = \frac{5d}{3w}.$$

For a normal braid tightness of the average knot (other than of the above type), use:

$$b = \frac{20d}{9w}.$$

For a very tightly braided knot whose bight edge is nowhere less than 2 under /2 over or vice-versa, use:

$$b = \frac{10d}{3w}.$$

Example 1: *Fan Knot coding as in Ex. 10 on page 75.*

This coding has a bight edge of 1 under/1 over and thus the formula $b = \frac{5d}{3w}$ has to be used.

Say that this fan knot has to fit over an object whose diameter is 10 mm (3/8"), and that it has to be made from a 2.5 mm (3/32") wide thong.

Then:

$$b = \frac{5 \times 10}{3 \times 2.5} = 6.7$$

Take $b = 7$

$$b = \frac{5 \times 3/8}{3 \times 3/32} = \frac{5 \times 3 \times 32}{3 \times 8 \times 3} = 6.7$$

Take $b = 7$

As has been said before, the thickness and rigidity of the thong plays a role in the relationship between the number of bights and the diameter of the knot and therefore it may well be possible that afer braiding the above knot with $b = 7$, the braid tightness is not as required. In this case we can change the value for b (i.e. one more or one less) as required. (N.B. remember that p and b cannot have a common factor other than 1).

Example 2: *Slow Helix Knot type I coding as in Ex.33 on page 79.*

Say that we require a normal braid tightness and thus we can use the formula $b = \frac{20d}{9w}$; (bight edge is other than 1 under/1 over).

Furthermore say that this knot has to fit over an object whose diameter is 21 mm (27/32"), and that it has to be made from a 3 mm (1/8") wide thong.

Then:

$$b = \frac{20 \times 21}{9 \times 3} = 15.6$$

Take $b = 15$

$$b = \frac{20 \times 27/32}{9 \times 1/8} = \frac{20 \times 27 \times 8}{9 \times 32 \times 1} = 15.0$$

Take $b = 15$

As before, if the braid tightness achieved with $b = 15$ is not as required, change the value of b accordingly. However, since $p = 22$, the nearest b values to $b = 15$ are $b = 13$ and $b = 17$; (p and b cannot have a common factor other than 1).

Example 3: *Two-pass Gaucho Knot coding as in Ex.11 on page 75.*

Say that we require a very tight braid and thus we should use the formula $b = \frac{10d}{3w}$; (bight edge is a 2 under/2 over.

Furthermore say that this knot has to fit over an object whose diameter is 7.5 mm (5/16"), and that it has to be made from a 3 mm (1/8") wide thong.

Then:

$$b = \frac{10 \times 7.5}{3 \times 3} = 8.3$$

Take $b = 8$

$$b = \frac{10 \times 5/16}{3 \times 1/8} = \frac{10 \times 5 \times 8}{3 \times 16 \times 1} = 8.3$$

Take $b = 8$

If we obtain with $b = 8$ a braid that is too tight, then we can decrease the number of bights to $b = 7$.

Solutions

1. Fan Knot (Botón Oriental) $p/b = 19/11$

1.	L→R	free run.
2.	R→L	u.
3.	L→R	u.
4.	R→L	2u–o.
5.	L→R	2u–o.
6.	R→L	u–o–u–o–u.
7.	L→R	u–o–u–o–u.
8.	R→L	u–o–2u–o–u.
9.	L→R	u–o–2u–o–u.
10.	R→L	o–u–o–3u–o–u.
11.	L→R	o–u–o–3u–o–u.
12.	R→L	o–2u–o–3u–2o–u.
13.	L→R	o–2u–o–3u–2o–u.
14.	R→L	o–2u–2o–3u–2o–2u.
15.	L→R	o–2u–2o–3u–2o–2u.
16.	R→L	o–2u–2o–u–o–2u–2o–2u.
17.	L→R	o–2u–2o–u–o–2u–2o–2u.
18.	R→L	2o–2u–2o–u–o–3u–2o–2u.
19.	L→R	2o–2u–2o–u–o–3u–2o–2u.
20.	R→L	2o–3u–2o–u–o–3u–3o–2u.
21.	L→R	2o–3u–2o–u–o–3u–3o–2u.
22.	R→L	2o–3u–3o–u–o–3u–3o–2u.

2. Turk's Head Knot $p/b = 8/5$

1.	L→R	free run.
2.	R→L	o.
3.	L→R	u.
4.	R→L	u–2o.
5.	L→R	o–2u.
6.	R→L	2u–2o.

7. L→R 2o–2u.
8. R→L o–2u–o–u–o.
9. L→R u–2o–u–o–u.
10. R→L o–u–o–u–o–u–o.

3. Turk's Head Knot $p/b = 5/8$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L o.
5. L→R o.
6. R→L o.
7. L→R o.
8. R→L 2o.
9. L→R 2o.
10. R→L 2o–u.
11. L→R 2o–u.
12. R→L 2o–u.
13. L→R 2o–u.
14. R→L o–u–o–u.
15. L→R o–u–o–u.
16. R→L o–u–o–u.

4. Turk's Head Knot $p/b = 13/21$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L o.
5. L→R o.
6. R→L o.
7. L→R o.
8. R→L u–o.
9. L→R u–o.
10. R→L u–o–u.
11. L→R u–o–u.
12. R→L u–o–u.
13. L→R u–o–u.
14. R→L u–2o–u.
15. L→R u–2o–u.
16. R→L u–2o–u.
17. L→R u–2o–u.
18. R→L 2u–2o–u.
19. L→R 2u–2o–u.
20. R→L 2u–2o–2u.
21. L→R 2u–2o–2u.
22. R→L 2u–2o–2u.
23. L→R 2u–2o–2u.
24. R→L 2u–3o–2u.

25. L→R 2u–3o–2u.
26. R→L 2u–3o–2u–o.
27. L→R 2u–3o–2u–o.
28. R→L 2u–3o–2u–o.
29. L→R 2u–3o–2u–o.
30. R→L 2u–2o–u–o–2u–o.
31. L→R 2u–2o–u–o–2u–o.
32. R→L 2u–2o–u–o–2u–o.
33. L→R 2u–2o–u–o–2u–o.
34. R→L u–o–u–2o–u–o–2u–o.
35. L→R u–o–u–2o–u–o–2u–o.
36. R→L u–o–u–2o–u–o–u–o–u–o.
37. L→R u–o–u–2o–u–o–u–o–u–o.
38. R→L u–o–u–2o–u–o–u–o–u–o.
39. L→R u–o–u–2o–u–o–u–o–u–o.
40. R→L u–o–u–o–u–o–u–o–u–o–u–o.
41. L→R u–o–u–o–u–o–u–o–u–o–u–o.
42. R→L u–o–u–o–u–o–u–o–u–o–u–o.

5. Turk's Head Knot $p/b = 8/13$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L o.
5. L→R u.
6. R→L o.
7. L→R u.
8. R→L u–o.
9. L→R o–u.
10. R→L u–2o.
11. L→R o–2u.
12. R→L u–2o.
13. L→R o–2u.
14. R→L 2u–2o.
15. L→R 2o–2u.
16. R→L 2u–2o.
17. L→R 2o–2u.
18. R→L o–2u–2o.
19. L→R u–2o–2u.
20. R→L o–2u–o–u–o.
21. L→R u–2o–u–o–u.
22. R→L o–2u–o–u–o.
23. L→R u–2o–u–o–u.
24. R→L o–u–o–u–o–u–o.
25. L→R u–o–u–o–u–o–u.
26. R→L o–u–o–u–o–u–o.

6. Turk's Head Knot $p/b = 13/8$

1. L→R free run.
2. R→L o.
3. L→R o.
4. R→L u–o–u.
5. L→R u–o–u.
6. R→L u–2o–u.
7. L→R u–2o–u.
8. R→L 2u–2o–2u.
9. L→R 2u–2o–2u.
10. R→L 2u–3o–2u–o.
11. L→R 2u–3o–2u–o.
12. R→L 2u–2o–u–o–2u–o.
13. L→R 2u–2o–u–o–2u–o.
14. R→L u–o–u–2o–u–o–u–o–u–o.
15. L→R u–o–u–2o–u–o–u–o–u–o.
16. R→L u–o–u–o–u–o–u–o–u–o–u–o.

7. Turk's Head Knot $p/b = 21/13$

1. L→R free run.
2. R→L o.
3. L→R o.
4. R→L 2o–u.
5. L→R 2o–u.
6. R→L o–u–o–u.
7. L→R o–u–o–u.
8. R→L u–o–u–2o–u.
9. L→R u–o–u–2o–u.
10. R→L u–2o–u–2o–2u.
11. L→R u–2o–u–2o–2u.
12. R→L u–2o–2u–2o–2u.
13. L→R u–2o–2u–2o–2u.
14. R→L 2u–2o–2u–3o–2u.
15. L→R 2u–2o–2u–3o–2u.
16. R→L 2u–3o–2u–3o–2u.
17. L→R 2u–3o–2u–3o–2u.
18. R→L o–2u–3o–2u–o–u–2o–2u.
19. L→R o–2u–3o–2u–o–u–2o–2u.
20. R→L o–2u–o–u–2o–2u–o–u–2o–u–o–u.
21. L→R o–2u–o–u–2o–2u–o–u–2o–u–o–u.
22. R→L o–2u–o–u–2o–u–o–u–o–u–2o–u–o–u.
23. L→R o–2u–o–u–2o–u–o–u–o–u–2o–u–o–u.
24. R→L o–u–o–u–o–u–2o–u–o–u–o–u–o–u–o–u–o–u.
25. L→R o–u–o–u–o–u–2o–u–o–u–o–u–o–u–o–u–o–u.
26. R→L o–u–o–u–o–u–o–u–o–u–o–u–o–u–o–u–o–u–o–u.

8. Two-pass Head Hunter's Knot $p/b = 11/8$

1. L→R free run.
2. R→L u.
3. L→R o.
4. R→L o–u.
5. L→R u–o.
6. R→L 2o–u–o.
7. L→R 2u–o–u.
8. R→L 2o–2u–o.
9. L→R 2u–2o–u.
10. R→L o–u–o–2u–o.
11. L→R u–o–u–2o–u.
12. R→L 2o–u–o–2u–2o.
13. L→R 2u–o–u–2o–2u.
14. R→L 2o–u–2o–2u–2o.
15. L→R 2u–o–2u–2o–2u.
16. R→L 2o–2u–2o–2u–2o.

9. Three-pass Gaucho Knot $p/b = 13/8$

1. L→R free run.
2. R→L o.
3. L→R o.
4. R→L 2o–u.
5. L→R 2o–u.
6. R→L o–u–o–u.
7. L→R o–u–o–u.
8. R→L 2o–u–2o–u.
9. L→R 2o–u–2o–u.
10. R→L 2o–2u–2o–2u.
11. L→R 2o–2u–2o–2u.
12. R→L 2o–2u–3o–2u.
13. L→R 2o–2u–3o–2u.
14. R→L 3o–2u–3o–3u.
15. L→R 3o–2u–3o–3u.
16. R→L 3o–3u–3o–3u.

10. Fan Knot (Botón Oriental) $p/b = 11/7$

1. L→R free run.
2. R→L u.
3. L→R u.
4. R→L 3u.
5. L→R 3u.
6. R→L 4u.
7. L→R 4u.
8. R→L 4u–o–u.
9. L→R 4u–o–u.
10. R→L 2u–o–2u–o–u.

11. L→R 2u–o–2u–o–u.
12. R→L o–2u–o–2u–2o–u.
13. L→R o–2u–o–2u–2o–u.
14. R→L o–2u–2o–2u–2o–u.

11. Two-pass Gaucho Knot $p/b = 13/8$

1. L→R free run.
2. R→L u.
3. L→R u.
4. R→L 3u.
5. L→R 3u.
6. R→L u–o–2u.
7. L→R u–o–2u.
8. R→L o–u–o–u–o–u.
9. L→R o–u–o–u–o–u.
10. R→L o–2u–o–u–o–2u.
11. L→R o–2u–o–u–o–2u.
12. R→L o–2u–o–2u–o–2u.
13. L→R o–2u–o–2u–o–2u.
14. R→L 2o–2u–o–2u–2o–2u.
15. L→R 2o–2u–o–2u–2o–2u.
16. R→L 2o–2u–2o–2u–2o–2u.

12. Turk's Head Knot $p/b = 12/7$

1. L→R free run.
2. R→L o.
3. L→R u.
4. R→L u–2o.
5. L→R o–2u.
6. R→L 2u–3o.
7. L→R 2o–3u.
8. R→L 3u–3o.
9. L→R 3o–3u.
10. R→L o–3u–o–u–2o.
11. L→R u–3o–u–o–2u.
12. R→L o–u–o–2u–o–u–o–u–o.
13. L→R u–o–u–2o–u–o–u–o–u.
14. R→L o–u–o–u–o–u–o–u–o–u–o.

13. Turk's Head Knot $p/b = 6/11$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L o.
5. L→R u.
6. R→L o.

7. L→R u.
8. R→L u–o.
9. L→R o–u.
10. R→L u–o.
11. L→R o–u.
12. R→L o–u–o.
13. L→R u–o–u.
14. R→L o–u–o.
15. L→R u–o–u.
16. R→L u–o–u–o.
17. L→R o–u–o–u.
18. R→L u–o–u–o.
19. L→R o–u–o–u.
20. R→L o–u–o–u–o.
21. L→R u–o–u–o–u.
22. R→L o–u–o–u–o.

14. Turk's Head Knot $p/b = 7/4$

1. L→R free run.
2. R→L o.
3. L→R o.
4. R→L u–o–u.
5. L→R u–o–u.
6. R→L u–2o–u–o.
7. L→R u–2o–u–o.
8. R→L u–o–u–o–u–o.

15. Two-pass Ring Knot $p/b = 5/7$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L u.
5. L→R u.
6. R→L u–o.
7. L→R u–o.
8. R→L u–o.
9. L→R u–o.
10. R→L 2u–o.
11. L→R 2u–o.
12. R→L 2u–2o.
13. L→R 2u–2o.
14. R→L 2u–2o.

16. Two-pass Ring Knot $p/b = 5/3$

1. L→R free run.
2. R→L o.
3. L→R o.
4. R→L u–2o.
5. L→R u–2o.
6. R→L 2u–2o.

17. Two-pass Ring Knot $p/b = 5/8$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L o.
5. L→R o.
6. R→L o.
7. L→R o.
8. R→L u–o.
9. L→R u–o.
10. R→L u–2o.
11. L→R u–2o.
12. R→L u–2o.
13. L→R u–2o.
14. R→L 2u–2o.
15. L→R 2u–2o.
16. R→L 2u–2o.

18. Four-pass Ring Knot $p/b = 9/10$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L u.
5. L→R u.
6. R→L 2u.
7. L→R 2u.
8. R→L 3u.
9. L→R 3u.
10. R→L 4u.
11. L→R 4u.
12. R→L 4u–o.
13. L→R 4u–o.
14. R→L 4u–2o.
15. L→R 4u–2o.
16. R→L 4u–3o.
17. L→R 4u–3o.
18. R→L 4u–4o.
19. L→R 4u–4o.
20. R→L 4u–4o.

19. Three-pass Ring Knot $p/b = 7/10$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L u.
5. L→R u.
6. R→L u–o.
7. L→R u–o.
8. R→L u–o.
9. L→R u–o.
10. R→L 2u–o.
11. L→R 2u–o.
12. R→L 2u–2o.
13. L→R 2u–2o.
14. R→L 2u–2o.
15. L→R 2u–2o.
16. R→L 3u–2o.
17. L→R 3u–2o.
18. R→L 3u–3o.
19. L→R 3u–3o.
20. R→L 3u–3o.

20. Two-pass Head Hunter's Knot $p/b = 7/5$

1. L→R free run.
2. R→L o.
3. L→R u.
4. R→L u–o.
5. L→R o–u.
6. R→L o–u–2o.
7. L→R u–o–2u.
8. R→L o–2u–2o.
9. L→R u–2o–2u.
10. R→L 2o–2u–2o.

21. Two-pass Head Hunter's Knot $p/b = 7/12$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L o.
5. L→R u.
6. R→L o.
7. L→R u.
8. R→L u–o.
9. L→R o–u.
10. R→L u–o.
11. L→R o–u.
12. R→L o–u–o.

13. L→R u–o–u.
14. R→L o–u–2o.
15. L→R u–o–2u.
16. R→L o–u–2o.
17. L→R u–o–2u.
18. R→L o–2u–2o.
19. L→R u–2o–2u.
20. R→L o–2u–2o.
21. L→R u–2o–2u.
22. R→L 2o–2u–2o.
23. L→R 2u–2o–2u.
24. R→L 2o–2u–2o.

22. Two-pass Head Hunter's Knot $p/b = 7/4$

1. L→R free run.
2. R→L u.
3. L→R o.
4. R→L o–u–o.
5. L→R u–o–u.
6. R→L 2o–u–2o.
7. L→R 2u–o–2u.
8. R→L 2o–2u–2o.

23. Two-pass Gaucho Knot $p/b = 9/5$

1. L→R free run.
2. R→L o.
3. L→R o.
4. R→L 3o.
5. L→R 3o.
6. R→L 4o–u.
7. L→R 4o–u.
8. R→L 2o–u–2o–2u.
9. L→R 2o–u–2o–2u.
10. R→L 2o–2u–2o–2u.

24. Botón Oriental $p/b = 11/6$

1. L→R free run.
2. R→L u.
3. L→R u.
4. R→L o–2u.
5. L→R o–2u.
6. R→L o–3u–o.
7. L→R o–3u–o.
8. R→L o–4u–2o.
9. L→R o–4u–2o.
10. R→L o–2u–o–2u–2o–u.

11. L→R o–2u–o–2u–2o–u.
12. L→R o–2u–2o–2u–2o–u.

25. Botón Oriental $p/b = 11/13$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L u.
5. L→R u.
6. R→L u–o.
7. L→R u–o.
8. R→L u–o–u.
9. L→R u–o–u.
10. R→L u–o–2u.
11. L→R u–o–2u.
12. R→L u–o–2u–o.
13. L→R u–o–2u–o.
14. R→L u–o–2u–o.
15. L→R u–o–2u–o.
16. R→L 2u–o–2u–o.
17. L→R 2u–o–2u–o.
18. R→L 2u–2o–2u–o.
19. L→R 2u–2o–2u–o.
20. R→L 2u–3o–2u–o.
21. L→R 2u–3o–2u–o.
22. R→L 2u–3o–3u–o.
23. L→R 2u–3o–3u–o.
24. R→L 2u–3o–3u–2o.
25. L→R 2u–3o–3u–2o.
26. R→L 2u–3o–3u–2o.

26. Botón Oriental $p/b = 11/9$

1. L→R free run.
2. R→L u.
3. L→R u.
4. R→L o–u.
5. L→R o–u.
6. R→L u–o–u.
7. L→R u–o–u.
8. R→L 2u–o–u.
9. L→R 2u–o–u.
10. R→L o–2u–o–2u.
11. L→R o–2u–o–2u.
12. R→L o–2u–2o–2u.
13. L→R o–2u–2o–2u.
14. R→L o–2u–3o–2u.
15. L→R o–2u–3o–2u.
16. R→L o–3u–3o–2u.

17. L→R o–3u–3o–2u.
18. R→L 2o–3u–3o–2u.

27. Ginfer Knot $p/b = 9/11$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L u.
5. L→R u.
6. R→L u–o.
7. L→R u–o.
8. R→L u–2o.
9. L→R u–2o.
10. R→L u–3o.
11. L→R u–3o.
12. R→L u–3o.
13. L→R u–3o.
14. R→L 2u–3o.
15. L→R 2u–3o.
16. R→L 3u–3o.
17. L→R 3u–3o.
18. R→L 3u–o–u–2o.
19. L→R 3u–o–u–2o.
20. R→L 3u–o–u–3o.
21. L→R 3u–o–u–3o.
22. R→L 3u–o–u–3o.

28. Samuel Knot $p/b = 8/13$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L u.
5. L→R o.
6. R→L u.
7. L→R o.
8. R→L 2u.
9. L→R 2o.
10. R→L 2u–o.
11. L→R 2o–u.
12. R→L 2u–o.
13. L→R 2o–u.
14. R→L u–o–u–o.
15. L→R o–u–o–u.
16. R→L u–o–u–o.
17. L→R o–u–o–u.
18. R→L o–u–o–u–o.
19. L→R u–o–u–o–u.
20. R→L o–u–o–2u–o.

21. L→R u–o–u–2o–u.
22. R→L o–u–o–2u–o.
23. L→R u–o–u–2o–u.
24. R→L o–2u–o–2u–o.
25. L→R u–2o–u–2o–u.
26. R→L o–2u–o–2u–o.

29. Aztec-Fan Knot $p/b = 15/8$
(The right hand side of this knot closes completely and gives the knot a flat top)

1. L→R free run.
2. R→L o.
3. L→R o.
4. R→L 3o.
5. L→R 3o.
6. R→L 4o–u.
7. L→R o–u–2o–u.
8. R→L 5o–2u.
9. L→R o–2u–2o–u–o.
10. R→L 3o–u–2o–2u–o.
11. L→R o–2u–3o–u–o–u.
12. R→L 3o–u–3o–2u–2o.
13. L→R o–2u–4o–u–o–2u.
14. R→L 3o–u–o–u–2o–2u–2o–u.
15. L→R o–2u–2o–u–2o–u–o–3u.
16. R→L 3o–u–o–2u–2o–2u–2o–u.

30. Perfect Three-Pass Herringbone Knot $p/b = 17/12$

1. L→R free run.
2. R→L o.
3. L→R u.
4. R→L o–u.
5. L→R 2o.
6. R→L 4u.
7. L→R 2o–u–o.
8. R→L u–o–u–o–u.
9. L→R 2u–o–2u.
10. R→L o–u–2o–u–o–u.
11. L→R 2u–o–u–o–2u.
12. R→L u–o–u–2o–2u–o.
13. L→R o–u–2o–2u–o–u.
14. R→L 2u–2o–u–2o–2u.
15. L→R u–o–2u–2o–2u–o.
16. R→L 2o–2u–2o–2u–2o–u.
17. L→R o–2u–2o–u–3o–2u.
18. R→L 2u–o–3u–2o–3u–o.
19. L→R 2o–2u–3o–2u–2o–u.
20. R→L o–3u–2o–2u–3o–3u.
21. L→R 2u–2o–3u–2o–3u–2o.

22. R→L 3o–2u–3o–3u–3o–u.
23. L→R o–3u–2o–3u–3o–3u.
24. R→L 2u–3o–3u–3o–3u–2o.

31. Perfect Three-Pass Herringbone Knot $p/b = 11/9$

1. L→R free run.
2. R→L o.
3. L→R u.
4. R→L 2u.
5. L→R u–o.
6. R→L u–o–u.
7. L→R o–2u.
8. R→L o–2u–o.
9. L→R 2u–o–u.
10. R→L 2u–o–2u–o.
11. L→R u–o–2u–2o.
12. R→L o–2u–2o–2u.
13. L→R 2u–o–3u–o.
14. R→L u–o–3u–3o.
15. L→R o–2u–3o–2u.
16. R→L 2u–3o–3u–o.
17. L→R u–2o–3u–3o.
18. R→L 2o–3u–3o–2u.

32. Beverley Knot $p/b = 13/9$

1. L→R free run.
2. R→L o.
3. L→R o.
4. R→L u–o.
5. L→R o–u.
6. R→L 2o–2u.
7. L→R u–2o–u.
8. R→L u–3o–u.
9. L→R o–u–2o–u.
10. R→L o–3u–2o–u.
11. L→R o–u–o–2u–2o.
12. R→L 3o–u–o–2u–o.
13. L→R u–2o–2u–o–u–o.
14. R→L 2u–3o–u–o–u–2o.
15. L→R o–u–o–u–3o–2u–o.
16. R→L o–u–o–2u–o–u–2o–2u.
17. L→R o–2u–o–u–o–2u–2o–u.
18. R→L 2o–2u–o–u–2o–2u–o–u.

33. Slow Helix Knot Type I $p/b = 22/15$
(Braid over Collar)

1. L→R free run.
2. R→L o.
3. L→R u.
4. R→L 2u.
5. L→R o-u.
6. R→L o-2u-o.
7. L→R o-u-2o.
8. R→L 2o-2u-o.
9. L→R o-u-3o.
10. R→L 2u-o-u-o-2u.
11. L→R 2u-3o-2u.
12. R→L u-o-2u-2o-u-o.
13. L→R u-3o-3u-o.
14. R→L u-o-2u-o-3u-o-u.
15. L→R 4o-3u-3o.
16. R→L o-3u-o-u-o-u-o-2u.
17. L→R 2o-3u-5o-u.
18. R→L o-u-o-u-o-2u-o-u-o-u-o-u.
19. L→R o-2u-o-u-5o-2u-o.
20. R→L 2u-o-u-o-u-o-2u-o-2u-o-u.
21. L→R 2u-5o-2u-o-2u-2o.
22. R→L u-o-2u-o-2u-o-u-2o-u-o-2u-o.
23. L→R u-3o-u-2o-2u-o-2u-3o-u.
24. R→L u-o-u-o-2u-o-2u-o-3u-o-u-o-u.
25. L→R 3o-2u-o-2u-3o-u-3o-2u.
26. R→L o-2u-o-u-o-u-o-u-o-3u-o-2u-o-u-o.
27. L→R 2o-2u-2o-2u-3o-u-3o-2u-2o.
28. R→L o-u-o-2u-2o-2u-o-u-o-2u-o-u-o-2u-o.
29. L→R o-2u-3o-u-3o-2u-3o-2u-3o.
30. R→L 2u-o-u-o-2u-o-u-o-2u-o-u-o-2u-o-u-o-u.

34. Slow Helix Knot Type I $p/b = 17/10$
(Braid over collar)

1. L→R free run.
2. R→L u.
3. L→R u.
4. R→L 3u.
5. L→R u-o-u.
6. R→L 2o-u-2o.
7. L→R u-2o-u-o.
8. R→L o-2u-2o-u.
9. L→R o-u-3o-u.
10. R→L o-3u-2o-2u.
11. L→R 2o-u-o-u-2o-u.

12. R→L o–3u–o–u–o–3u.
13. L→R 2o–u–o–2u–2o–u–o.
14. R→L 3u–o–u–o–4u–o.
15. L→R o–u–2o–2u–2o–u–2o.
16. R→L o–u–o–u–2o–2u–o–u–o–u–o.
17. L→R o–2u–2o–u–3o–2u–2o.
18. R→L 2u–o–u–o–u–o–u–o–2u–o–u–o–u.
19. L→R 2u–3o–u–3o–2u–3o–u.
20. R→L u–o–2u–o–u–o–2u–o–u–o–2u–o–u.

35. Fast Helix Knot $p/b = 17/10$
(Braid over collar)

1. L→R free run.
2. R→L u.
3. L→R o.
4. R→L 3o.
5. L→R u–o–u.
6. R→L 5u.
7. L→R o–2u–o–u.
8. R→L 2o–2u–2o.
9. L→R u–3o–u–o.
10. R→L o–3u–2o–2u.
11. L→R 3o–2u–3o.
12. R→L 3u–2o–4u–o.
13. L→R u–2o–u–2o–u–2o–u.
14. R→L u–2o–u–o–u–o–u–2o–u.
15. L→R o–u–2o–2u–2o–u–2o.
16. R→L u–o–4u–o–2u–o–3u.
17. L→R u–2o–2u–2o–2u–2o–2u.
18. R→L o–u–o–2u–2o–2u–o–u–o–2u–o.
19. L→R o–2u–3o–u–3o–2u–3o.
20. R→L u–o–2u–o–u–o–2u–o–u–o–2u–o–u.

36. Basket Weave Knot $p/b = 17/15$
Type $6n + 5$

1. L→R free run.
2. R→L o.
3. L→R u.
4. R→L o–u.
5. L→R 2o.
6. R→L 3u.
7. L→R 2o–u.
8. R→L o–u–2o.
9. L→R 4u.
10. R→L o–u–2o–u.
11. L→R 2o–u–2o.
12. R→L 6u.
13. L→R 2o–u–2o–u.

14. R→L o–u–2o–u–2o.
15. L→R 7u.
16. R→L o–u–2o–u–2o–2u.
17. L→R 2o–u–2o–u–2o–u.
18. R→L 7u–o–u–o.
19. L→R o–u–2o–u–2o–3u.
20. R→L 2o–u–2o–2u–3o–u.
21. L→R 6u–o–u–o–u–o.
22. R→L o–u–2o–2u–3o–3u.
23. L→R 2o–u–2o–3u–3o–u.
24. R→L 4u–o–u–o–u–o–u–o–u–o.
25. L→R o–u–2o–3u–3o–3u.
26. R→L 2o–2u–3o–3u–3o–u.
27. L→R 3u–o–u–o–u–o–u–o–u–o–u–o.
28. R→L o–2u–3o–3u–3o–3u.
29. L→R 2o–3u–3o–3u–3o–u.
30. R→L u–o–u–o–u–o–u–o–u–o–u–o–u–o–u–o.

37. Basket Weave Knot $p/b = 14/15$
Type $6n + 2$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L u.
5. L→R u.
6. R→L 2o.
7. L→R o–u.
8. R→L o–2u.
9. L→R 3u.
10. R→L u–o–u–o.
11. L→R u–3o.
12. R→L 3o–2u.
13. L→R o–u–o–u–o.
14. R→L o–3u–2o.
15. L→R 3u–3o.
16. R→L u–o–u–o–u–o–u.
17. L→R u–3o–3u.
18. R→L 3o–3u–2o.
19. L→R o–u–o–u–o–u–o–u.
20. R→L o–3u–3o–2u.
21. L→R 3u–3o–3u.
22. R→L u–o–u–o–u–o–u–o–u–o.
23. L→R u–3o–3u–3o.
24. R→L 3o–3u–3o–2u.
25. L→R o–u–o–u–o–u–o–u–o–u–o.
26. R→L o–3u–3o–3u–2o.
27. L→R 3u–3o–3u–3o.
28. R→L u–o–u–o–u–o–u–o–u–o–u–o–u.

29. L→R u–3o–3u–3o–3u.
30. R→L 3o–3u–3o–3u–o.

38. O1 Knot $p/b = 17/10$

1. L→R free run.
2. R→L u.
3. L→R o.
4. R→L 3u.
5. L→R o–u–o.
6. R→L o–u–2o–u.
7. L→R 2u–o–2u.
8. R→L u–2o–2u–o.
9. L→R 2o–u–3o.
10. R→L o–3u–2o–2u.
11. L→R o–u–2o–u–o–u–o.
12. R→L u–o–u–o–3u–o–u–o.
13. L→R 3o–2u–4o–u.
14. R→L o–4u–o–u–o–3u.
15. L→R u–o–u–3o–2u–o–u–o.
16. R→L 2u–2o–u–o–u–o–2u–2o–u.
17. L→R u–o–u–o–2u–2o–u–o–u–o–u.
18. R→L o–2u–o–u–o–u–o–u–o–2u–o–u–o.
19. L→R o–u–o–u–3o–u–2o–u–o–u–2o.
20. R→L u–o–2u–o–u–o–2u–o–u–o–2u–o–u.

39. O2 Knot $p/b = 17/14$

1. L→R free run.
2. R→L u.
3. L→R o.
4. R→L 2u.
5. L→R 2o.
6. R→L 2u–o.
7. L→R o–u–o.
8. R→L u–o–u–o.
9. L→R u–o–2u.
10. R→L o–u–4o.
11. L→R o–4u–o.
12. R→L 4o–u–2o.
13. L→R 5u–o–u.
14. R→L 4o–u–3o.
15. L→R 2u–o–3u–o–u.
16. R→L o–u–3o–u–o–u–o.
17. L→R o–2u–o–u–o–u–2o.
18. R→L 2o–u–o–u–o–3u–o.
19. L→R o–u–o–u–3o–u–2o.
20. R→L o–u–o–3u–o–2u–2o–u.
21. L→R u–3o–u–2o–u–2o–u–o.
22. R→L 2u–o–2u–o–2u–o–u–o–2u.

23. L→R u−2o−u−3o−u−2o−2u−o.
24. R→L u−o−3u−o−2u−2o−u−o−u−o.
25. L→R 3o−u−o−2u−o−u−o−2u−2o.
26. R→L u−o−u−2o−u−o−u−2o−2u−o−u−o.
27. L→R o−u−o−u−o−2u−2o−u−o−u−2o−u.
28. R→L u−o−u−2o−2u−o−u−o−2u−2o−u−o.

40. O5 Knot $p/b = 20/21$

1. L→R free run.
2. R→L free run.
3. L→R free run.
4. R→L o.
5. L→R u.
6. R→L o−u.
7. L→R u−o.
8. R→L o−u−o.
9. L→R u−2o.
10. R→L o−u−o−u.
11. L→R u−o−2u.
12. R→L o−u−o−u−o.
13. L→R u−2o−2u.
14. R→L o−u−o−u−o−u.
15. L→R u−o−2u−2o.
16. R→L o−2u−2o−u−o.
17. L→R u−o−u−o−u−2o.
18. R→L o−u−2o−2u−o−u.
19. L→R u−o−u−o−u−o−2u.
20. R→L o−2u−2o−2u−2o.
21. L→R u−o−u−o−u−o−u−o−u.
22. R→L o−u−2o−2u−2o−2u.
23. L→R u−2o−2u−o−u−o−u−o.
24. R→L o−u−o−u−o−2u−2o−2u.
25. L→R u−o−2u−2o−u−o−u−o−u.
26. R→L o−u−o−u−o−u−2o−2u−2o.
27. L→R u−2o−2u−2o−2u−o−u−o.
28. R→L o−u−o−u−o−u−o−u−o−2u−2o.
29. L→R u−o−2u−2o−2u−2o−u−o−u.
30. R→L o−2u−2o−u−o−u−o−u−2o−2u.
31. L→R u−o−u−o−u−2o−2u−2o−2u−o.
32. R→L o−u−2o−2u−o−u−o−u−o−u−o−2u.
33. L→R u−o−u−o−u−o−2u−2o−2u−2o−u.
34. R→L o−2u−2o−2u−2o−u−o−u−o−u−2o.
35. L→R u−o−u−o−u−o−u−o−u−2o−2u−2o−u.
36. R→L o−u−2o−2u−2o−2u−o−u−o−u−o−u−o.
37. L→R u−2o−2u−o−u−o−u−o−2u−2o−2u−o.
38. R→L o−u−o−u−o−2u−2o−2u−2o−u−o−u−o−u.
39. L→R u−o−2u−2o−u−o−u−o−u−o−u−2o−2u−o.
40. R→L o−u−o−u−o−u−2o−2u−2o−2u−o−u−o−u−o.

41. L→R u–2o–2u–2o–2u–o–u–o–u–o–2u–2o–u.
42. R→L o–u–o–u–o–u–o–u–o–2u–2o–2u–2o–u–o.

41. O3 Knot $p/b = 16/9$

1. L→R free run.
2. R→L o.
3. L→R o.
4. R→L u–o–u.
5. L→R o–2u.
6. R→L u–o–u–2o.
7. L→R o–2u–o–u.
8. R→L 3u–2o–u–o.
9. L→R o–u–2o–3u.
10. R→L u–o–2u–o–u–2o.
11. L→R 3o–3u–o–u.
12. R→L o–u–o–u–o–2u–3o.
13. L→R u–o–u–2o–u–2o–2u.
14. R→L o–u–o–2u–2o–u–2o–u–o.
15. L→R u–2o–u–o–u–2o–u–o–2u.
16. R→L o–u–2o–3u–o–2u–2o–u–o.
17. L→R u–o–u–o–u–o–2u–o–2u–2o–u.
18. R→L o–2u–2o–u–2o–2u–o–u–o–u–o.

42. O4 Knot $p/b = 16/15$

1. L→R free run.
2. R→L o.
3. L→R u.
4. R→L u–o.
5. L→R o–u.
6. R→L o–u–o.
7. L→R u–o–u.
8. R→L o–2u–o.
9. L→R o–u–o–u.
10. R→L u–2o–u–o.
11. L→R u–o–u–o–u.
12. R→L u–o–u–o–u–o.
13. L→R o–2u–2o–u.
14. R→L o–u–o–u–o–u–o.
15. L→R u–2o–2u–o–u.
16. R→L u–o–u–o–u–o–u–o.
17. L→R u–2o–u–o–u–o–u.
18. R→L o–u–o–u–2o–2u–o.
19. L→R o–2u–o–u–o–u–o–u.
20. R→L u–o–u–o–2u–2o–u–o.
21. L→R o–u–o–u–o–u–o–u–o–u.
22. R→L o–u–2o–2u–o–u–o–u–o.
23. L→R u–o–u–o–u–o–2u–2o–u.
24. R→L u–o–2u–2o–u–o–u–o–u–o.

25. L→R o–u–o–u–o–u–2o–2u–o–u.
26. R→L 2o–2u–o–u–o–u–o–u–o–u–o.
27. L→R u–o–u–o–2u–2o–u–o–u–o–u.
28. R→L 2u–2o–u–o–u–o–u–2o–2u–o.
29. L→R o–u–o–u–2o–2u–o–u–o–u–o–u.
30. R→L o–u–o–u–o–u–o–u–o–2u–2o–u–o.

BIBLIOGRAPHY

[1] Encyclopaedia of Rawhide and Leather Braiding (1981). Bruce Grant; Cornell Maritime Press, P.O.Box 456, Centreville, Maryland 21617, U.S.A.

[2] The Ashley Book of Knots (1944). Clifford W. Ashley; Doubleday & Company Inc., 501 Franklin Avenue, Garden City, New York 11530, U.S.A.

[3] Leather Braiding (1981). Bruce Grant; Cornell Maritime Press, P.O.Box 456, Centreville, Maryland 21617, U.S.A.

[4] How to Make Cowboy Horse Gear (1982). Bruce Grant; Cornell Maritime Press, P.O.Box 456, Centreville, Maryland 21617, U.S.A.

[5] The Bushmen's Handcrafts (1981). R. M. Williams; R. M. Williams Pty. Ltd., 5 Percy Street, Prospect, South Australia.

[6] Encyclopaedia of Knots and Fancy Rope Work (1984). Raoul Graumont and John Hensel; Cornell Maritime Press, P.O.Box 456, Centreville, Maryland 21617, U.S.A.

[7] The Mathematical Theory of Knots and Braids (1983). S. Moran; North Holland Mathematics Studies Vol 82.

[8] A New Theory of Braiding: Research Report RR1/1 (1988). A. G. Schaake and J. C.Turner; Department of Mathematics and Statistics, University of Waikato, Hamilton, New Zealand.

[9] A New Theory of Braiding: Research Report RR1/2 (1988). A. G. Schaake and J. C.Turner; Department of Mathematics and Statistics, University of Waikato, Hamilton, New Zealand.

INDEX

E

F

G

H

I

M

N

O

P

R

S

T